Universal differential equations for infectious disease modelling

A common framework for modelling disease transmission is compartmental models that describe how the number of individuals in pre-determined compartments change over time using a set of coupled ordinary differential equations (ODEs). One example is the SIR-model, which divides the population into a Susceptible, Infected and Recovered compartment. The rate at which people become infected in this model depends on the rate at which people interact, known as the contact rate.

A particular difficulty of infectious disease modelling is that the contact rate is time-dependent and affected by e.g. public health recommendations and societal trends. One way of improving model accuracy is to include time series data that correlates with the contact rate, such as mobility data from mobile phones or data from public transport. In a previous article we have shown that this type of data can improve predictions of hospital admissions [1].

The standard approach for fitting compartmental models to data is to specify a functional form of the system of ODEs that depend on a number of parameters that are estimated by comparing the model output (e.g. hospital admissions/week) to the actual outcome. However, the functional forms are often poorly grounded in theory or observation.

A complementary approach is to make use of Universal Differential Equations (UDEs) where the right hand sides of the ODEs are represented using a deep neural network [4]. In this setting one does not have to specify how e.g. mobility data affects disease transmission. Instead this emerges as the neural network is trained on outcome data.

The aim of this PhD-project is to construct and analyse UDE models for infectious diseases. This will serve as a test bed for investigating open questions about UDEs, such as rates of convergence and quantification of uncertainty. The approach is related to non-parametric inference for ODEs [2], and comparisons will therefore be made with those type of models. For such models it has been shown that convergence can be greatly improved by using a probability measure adapted to the dynamics of the system [3]. We will therefore pursue similar questions and techniques in the context UDEs.

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