

# Critical Galton-Watson processes with overlapping generations

PhD project proposal by Serik Sagitov

The Galton-Watson process is a classical stochastic model of a self-replicating system, see [8] and [1]. Such models are shown to be useful for studying patterns of stochastic evolution of viruses and other biological populations, see [2], [3], and [4]. A Galton-Watson process is defined as a Markov chain  $Z(0), Z(1), Z(2), \dots$ , satisfying the branching property

$$Z(t+1) = Z_1(t) + \dots + Z_N(t),$$

where the summands are independent copies of  $Z(t)$  describing the daughter processes generated by  $N$  daughters of the ancestral individual.

The PhD project will focus on the critical reproduction case, when  $E(N) = 1$ . In fact, we consider a more general self-replicating system allowing for overlapping generations or *GWO-processes* for short. Our recent results [6], [7] dealt with the critical GWO-processes, assuming that both  $\text{Var}(N)$  and the mean generation length  $a$  are finite.

The purpose of the project is to relax the conditions  $\text{Var}(N) < \infty$ ,  $a < \infty$  and establish new limit theorems for the critical GWO-processes. One promising special class of GWO-processes, which may generate a new limit theorem for the age-dependent branching processes, is the one having the linear-fractional reproduction law, see [5].

## 1 References

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