The Hardy–Littlewood Circle Method, 7.5 hp

Course period:
November 2, 2020 - January 17, 2021

Last day for application:
November 2, 2020

Course leader / Address for applications:
Kirsti Biggs / biggs@chalmers.se

Course description (Advertisement for Ph.D. students):

The Hardy–Littlewood circle method was originally developed one hundred years ago, and has become a crucial tool within analytic number theory. It is used to count solutions to Diophantine equations and inequalities in a wide variety of contexts. The course will begin with an overview of the circle method and its history, followed by a more detailed treatment as given in chapters 2-6 of Davenport's book "Analytic Methods for Diophantine Equations and Inequalities". This will have a focus on the classical application to Waring's problem, a question concerning the representation of integers as sums of perfect powers.

Next, we will discuss several other important applications of the method, such as to the ternary Goldbach conjecture and other problems involving primes, and to Diophantine inequalities. I will also mention recent developments with connections to harmonic analysis (related to the resolution of the 'Main Conjecture' in Vinogradov's mean value theorem).

For assessment purposes, I will invite students to present a paper of their choice on a related topic, which will have the additional benefit of providing further examples of the method.

The course will run in the second reading period. The course will run twice a week (2 hours each time). The schedule will be decided with the participants' input. It will be possible to follow the course online.

Responsible department and other participation departments/organisations:
Mathematics Department

Teacher:
Kirsti Biggs

Examiner:
Kirsti Biggs

The Hardy–Littlewood Circle Method, 7.5 hp

1. Confirmation
The syllabus was confirmed by the Head of the Department of XXX 202X-XX-XX, 202X-XX-XX.

Disciplinary domain: Science
Department in charge: Department of Mathematical Sciences
Main field of study: Mathematics

2. Position in the educational system
Ph.D. course.

3. Entry requirements
The student should have knowledge of elementary number theory and basic analysis.

4. Course content
The aim of this course is to give an introduction to the Hardy–Littlewood circle method, its history and various applications.

Some of the topics which will be covered are: the circle method as applied to Waring's problem; Goldbach's conjecture and other problems involving primes; Diophantine inequalities; Vinogradov's mean value theorem and connections to harmonic analysis.

5. Outcomes
At the end of the course, the students will have acquired an understanding of the Hardy–Littlewood circle method, as well as knowledge concerning several of its key applications.

6. Literature
The primary book for the course is "Analytic Methods for Diophantine Equations and Inequalities" by H. Davenport. Further reading for the interested participant may be found in "The Hardy–Littlewood Method" by R. C. Vaughan.

7. Assessment
For assessment students will be asked to present a paper of their choice on a related topic, agreed in advance with the lecturer.

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

8. Grading scale
The grading scale comprises Fail (U), Pass (G).

9. Course Evaluation
The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

10. Language of instruction
The language of instruction is English.