Singular Integral Equations, 7.5 hp

Course period:
August 31 - October 31, 2020

Last day for application:
August 31, 2020

Course leader / Address for applications:
Andreas Rosén / rosenan@chalmers.se

Course description (Advertisement for Ph.D. students):

Linear equations appear for example when solving boundary value problems for linear PDEs, and typically the operator appearing at the boundary is singular in the sense that its kernel is like $|x-y|^{-d}$, where d is dimension of the boundary. The starting point for modern real harmonic analysis around 1980 was the Coifman-McIntosh-Meyer theorem and the $T1$ and Tb theorems, which give a complete L_2 understanding of general non-convolution singular integral operators (SIOs). During the course we will explore the beautiful wavelet and paraproduct structures hiding behind these results.

The above results aim to prove boundedness of SIOs. For applications to partial differential equations it is equally important to have tools to prove invertibility of linear equations involving SIOs. The second part of the course will be devoted to the theory of Fredholm operators on Banach spaces, done properly following Kato.

If time permits, we shall also look at the Beylkin-Coifman-Rokhlin numerical algorithm for wavelet discretizations of SIOs.

The course will run once a week (2 hours) during study period 1, some weeks twice. The schedule will be decided by participants at an introductory meeting.

Responsible department and other participation departments/organisations:
Mathematics Department

Teacher:
Andreas Rosén

Examiner:
Andreas Rosén

Singular Integral Equations, 7.5 hp

1. Confirmation

Disciplinary domain: Science
Department in charge: Department of Mathematical Sciences
Main field of study: Mathematics

2. Position in the educational system
Elective course; third-cycle education

3. Entry requirements
Fourier analysis, Functional Analysis

4. Course content
The course will cover a suitable subset of the following topics. The final curriculum will be decided upon during the course.

- Singular integrals and applications to partial differential equations.
- The $T(1)$ and $T(b)$ theorems and related harmonic analysis.
- Wavelet discretizations of singular integral operators.
- Perturbation and duality theory for Fredholm operators.

5. Outcomes
At the end of the course, the students will have acquired knowledge about some of the main results about and techniques for solving singular integral equations.

6. Required reading
The course will be based on lecture notes which cover the required reading. Lecture notes from an earlier version of the course are available at: `http://www.math.chalmers.se/~rosenan/FST.html`

7. Assessment
There will be a few homework sheets, and an oral exam at the end of the course.

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

8. Grading scale
The grading scale comprises Fail (U), Pass (G)

9. Course Evaluation
The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

10. Language of instruction
The language of instruction is English.