Summation formulas in analytic number theory

This project concerns one of the most important tools in analytic number theory, namely summation formulas. Particular attention will be paid to Euler-Maclaurin formula, Poisson summation formula and Voronoi summation formula. The main application will be to the study of summation formulas for the hyperbola and to Dirichlet's divisor problem. This problem has a long and rich history and concerns with investigation of one of the most "elementary" arithmetic functions in number theory - the divisor function d(n), which counts the number of positive divisors of a natural number "n". Dirichlet studied the asymptotic behaviour of the sum

$$D(N) = \sum_{n=1}^{N} d(n).$$

This sum has a geometric interpretation as the number of pairs of positive integers (m, n) with $m \cdot n \leq N$, i.e. corresponding to lattice points in the first quadrant located to the left of the hyperbola $x \cdot y = N$. In 1849, Dirichlet proved the formula

$$D(N) = N \cdot log(N) + (2\gamma - 1) \cdot N + O(N^{a})$$

with $a = 1/2$, where γ is the Euler-Mascheroni constant. Dirichlet's divisor problem consists of finding the smallest possible value of a, which is conjectured to be $1/4$. Despite the fact that many mathematicians have contributed to this problem (Voronoi, Sierpinski, Littlewood and Walfisz, van der Corput, Chen, Kolesnik, Vinogradov, Iwaniec and Mozzochi, Huxley, Bourgain and Watt) this is still open. The current record is by Bourgain and Watt (2017) who obtained $a = 517/1648$.
The expected outcome of the project is an account on the theory of summation formulas and the main techniques used in some of the results on Dirichlet's divisor problem. Furthermore, the project may include investigations on divisor problem in arithmetic progressions.