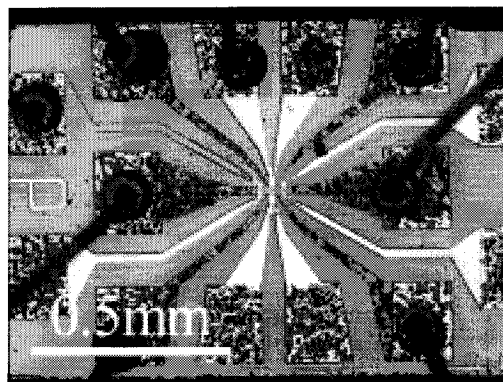
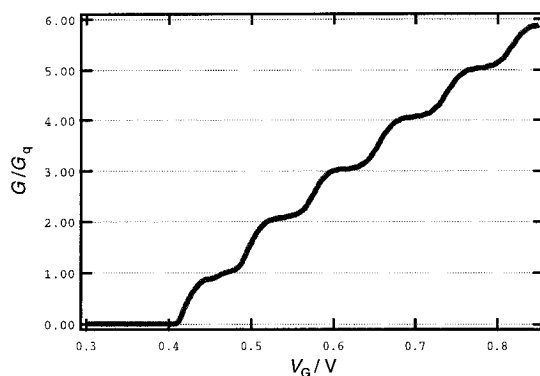


Me1

Quantized Conductance Stepping through the mesoscopic regime



Målsättning

Kvantiserad konduktans i en en-dimensionell kanal studeras vid låg temperatur. I en mycket smal ledare (kanal) får det bara plats en multipel av halva våglängden hos elektronens vågfunktion. Därför är konduktansen kvantiserad till multiplar av 'kvantkonduktansen' $2e^2/h$. Med hjälp av lockin-förstärkare mäts konduktansen hos en ca 50nm bred kanal vars bredd kan varieras med en extern spänning.

Förberedelser

Läs Lab-PM.

1. Introduction

Consider a piece of conductor. If it is only two dimensional the conductance of this conductor will only depend on the specific conductivity of the material, σ , the length L , and the width W of the conductor.

$$G = \sigma \frac{W}{L} \quad (1)$$

As W decreases the conductance approaches zero. If equation 1 was to be always true the conductance of any conductor would decrease linearly all the way down to zero. But as we will see in this lab the conductance of a very small conductor takes discrete values. At some size there must be a transition from this classical behavior into the quantum regime. To determine in which regime the

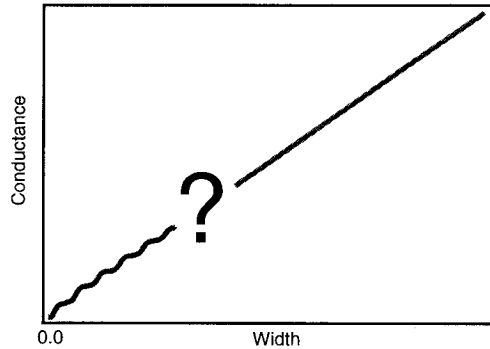


Figure 1: Conductance as a function of width.

conductance takes place we have to consider several length scales in our conductor. At first there are the physical dimensions of the conductor. In a conductor the current carrying electrons behave like waves and the wave length of the conductance electrons is called the Fermi wavelength λ_F . The mean free path l_e which is the distance the electron travels between elastic scattering events, and the coherence length l_ϕ , the length the electron travels before it is inelastically scattered. As long as the dimensions of the conductor are bigger than all three of these characteristic length scales the conductance will follow the ohmic relation in formula 1 (figure 1). If the dimensions of the conductor are smaller than the mean free path we say that the conductor is ballistic, i.e. electrons can pass through it without any scattering events at all (figure 1).

For the first time in 1965 Shavrin [2] studied the conductance in ballistic conductors. But due to the

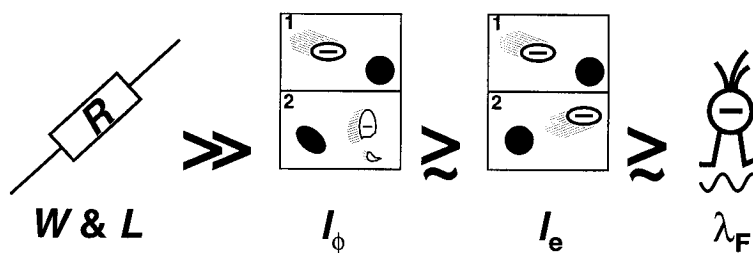


Figure 2: Comparison of relevant length scales in a classical conductor. The physical dimensions are much larger than the others. W and L are dimensions of the sample, l_ϕ the coherence length, l_e the electron mean free path, and λ_F the Fermi wave length.

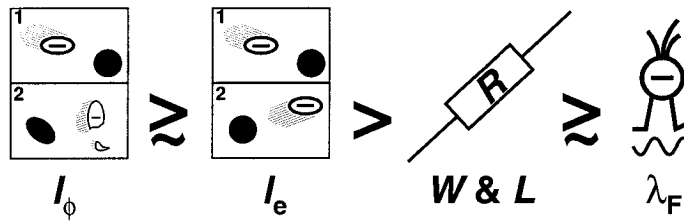


Figure 3: In a quantum conductor the physical dimensions are comparable to the Fermi wavelength.

very short Fermi wavelength in metals, about 1 Å, it is almost impossible to see these effect in well defined metallic samples. It was thus not until the knowledge and technology about semi-conductors was well enough developed that samples with the needed dimensions could be fabricated. As we will see in section 3 it is now possible to fabricate ballistic samples smaller than the Fermi wavelength in semiconductors ($\lambda_F \approx 40nm$).

It is this technology that allows us to demonstrate the radical change of behavior of the conductance as we move from the classical regime into this quantum regime.

2. Current quantization

Now let us take a closer look at at the conductance of our ‘wire’ as its dimensions are further and further reduced. Since we have already reasoned that the ohmic behavior given by formula 1 will not be valid below a certain size it is necessary to look at the wire in a different way. Imagine the wire to consist of two contacts connected by a ballistic conductor (see figure 4). Before continuing

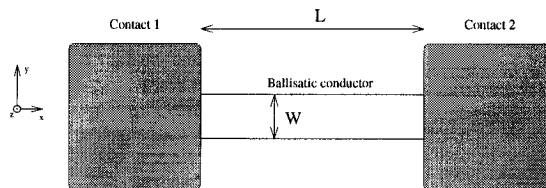


Figure 4: A ballistic conductor is sandwiched between two contacts.

let us reflect how the current is transported in the conductor. From quantum mechanics we are familiar with the ‘particle in a box’ problem. Solving the Schrödinger equation for such a particle in a box reveals that the particle can only occupy certain, quantized energy levels (figure 5), i.e. standing wave type solutions.

But where does the finite and quantized conductance come from? After all, the ballistic conductor should have no resistance since electrons can pass through it without colliding. The resistance arises from the interface between the contact and the conductor. In the large contact the current is carried by infinitely many ‘transverse modes’. In the ballistic conductor however, the current is carried by a limited number of modes, i.e. of parallel transport channels. This requires a redistribution of the current from the contact into the conductor and leads to the finite conductance.

To evaluate the effective resistance G^{-1} we consider a ballistic conductor with an applied bias

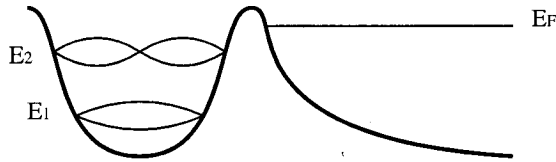


Figure 5: In a one-dimensional well the electron wave functions are standing waves with quantized energy levels. As a comparison, in the reservoir to the right electrons can occupy any energy level up to the Fermi level.

$V_{Bias} = (\mu_1 - \mu_2)/e$ ¹. We consider the interfaces to be reflection less, i.e. the electrons can pass from the conductor into the contacts with unit probability.

Next we are interested in how much current each modes, i.e. each channel through the ballistic conductor, can carry.

A uniform electron gas with n electrons, of charge e , per unit length, moving with a velocity v carries a current equal to env . A plain wave in one dimension is given by $\Psi = e^{-ikx}$ where k is the wave number. To obtain the current carried by a single mode we have to sum over all the contributions from the electron waves in this modes having different k -values. The distribution of these waves is denoted by some function which we will call f .

The electron density in a conductor of length L carried by a single electron wave is $1/L$. Thus we can write the current carried by a single mode as:

$$I = \frac{e}{L} \sum_k v f \quad (2)$$

using the relation $v = \frac{1}{\hbar} \frac{\partial E}{\partial k}$ and converting the sum to an integral² we get:

$$I = \frac{2e}{h} \int f dE \quad (3)$$

The integral is equal to the energy difference between the first and the second contact, i.e. $\mu_1 - \mu_2$. Using $V_{Bias} = (\mu_1 - \mu_2)/e$ we can write the current carried by a single mode as:

$$I = \frac{2e^2}{h} V_{Bias} \quad (4)$$

If we denote the number of conductance channels in the conductor by m , we get the total current through the conductor as:

$$I = \frac{2e^2}{h} m V_{Bias} \quad (5)$$

and thus the conductance is:

$$G_c = \frac{2e^2}{h} m \quad (6)$$

Since it is the wave nature of the electrons that determines the conductance we should really be speaking about a ballistic waveguide. We can now see that the resistance of this ballistic waveguide is:

$$G_c^{-1} = \frac{h}{2e^2 m} \approx \frac{12.9k\Omega}{m} \quad (7)$$

¹ $\mu_{1,2}$ is the chemical potential at the left and right contact respectively. It is defined as the electric potential divided by the electron charge.

² $\sum_k \rightarrow 2(\text{for spin}) \times \frac{L}{2\pi} \int dk$

The contact resistance in a single moded conductor is $\sim 12.9k\Omega$, which is certainly not negligible! Usually we are concerned with conductors with huge numbers of modes and thus the total contact resistance is of no importance. In our example however the width will be so small that the number of channels is reduces to a point where the channel resistance determines the conductance of the sample.

It is therefore important to estimate the number of channels in the sample. Recalling the analogy to the ‘particle-in-a-box’ problem, we assume periodic boundary conditions, and can thus estimate the number of channels as:

$$m = \text{Int}\left[\frac{W}{\lambda_F/2}\right] \quad (8)$$

where $\text{Int}[x]$ represents x rounded to the next smaller integer. This is analog to the standing waves in the ‘particle-in-a-box’ (c.f. figure 5) where only waves which can fit an integer number of half-wavelength inside the box, are allowed. Practically this means that a conductor of half the Fermi wavelength width will have one conducting channel.

In our sample we can change the effective width of our conductor by applying a voltage to the gates. As we will see in section 3 the electric field resulting from the gate voltage will allow us to change the effective width of the conductor. By changing the gate voltage we can thus choose m .

3. The Sample

In the previous section we have seen what to expect from a very small conductor. But are we able to verify these predictions experimentally? To do this we need a sample that has dimensions of the Fermi wavelength in the chosen material. Metals will not be suitable since the Fermi wavelength in metals is around 1\AA , i.e. of the same order of magnitude as atoms. However in semiconductors, the Fermi wavelength is a few tenth of nanometers, thus the choice of a semiconductor sample.

As we have seen in section 2 the conductor should be two dimensional to start with. Than we would like to further reduce the width of the conductor down to a size comparable to the Fermi wavelength.

The two dimensionality is achieved by layering different semiconductor materials on top of each

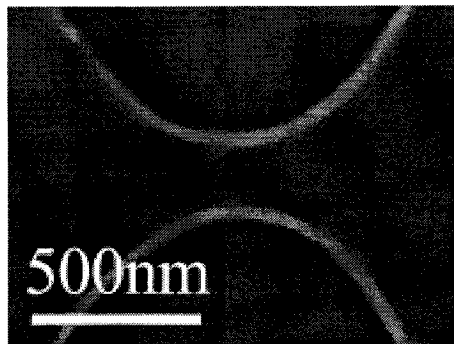


Figure 6: A SEM picture of the sample. The constriction is approx. 250 nm wide.

other. In our case this is a GaAs-AlGaAs heterostructure. Due to the matching of the energy levels in the two layers a very thin region is formed at the interface in which electrons can move with great ease. This is called a two dimensional electron gas (2-DEG) since the electrons are restricted to two dimensional movement. Within this two dimensional plane however they are comparatively free to

move which results in a long mean free path. Such 2-DEGs are also used on an industrial scale in High Electron Mobility Transistors (HEMT) for high frequency applications, e.g. for mobile communication.

To be able to vary the width of the conductor we apply a voltage to the gate which has been fabricated on the sample. The charge that accumulates on the gate will create an electric field which penetrates the 2-DEG. Since the electrons in the 2-DEG are so free to move they will be strongly affected by the field effect from the gate. Depending on the sign of the gate voltage we can either raise or lower the electron density in the 2-DEG. As we will see in the experiment our sample is non conducting if no gate voltage is applied. This is called an enrichment type 2-DEG since we will have to apply a positive gate voltage in order to attract electrons into the 2-DEG, thus enriching the 2-DEG with electrons.

Using this gate technique we can control the effective width of the conductor, thus controlling how

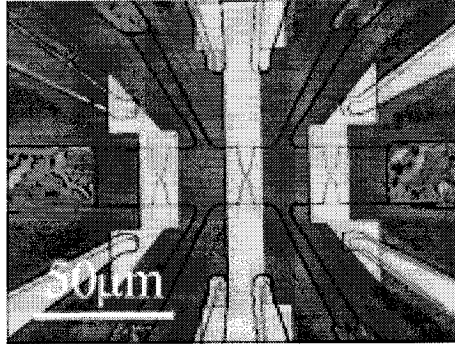


Figure 7: The picture shows three constrictions, the gates and the leads connecting the structures.

many channel will be able to carry a current. This should enable us to measure the effect of the contact resistance on the conductance of the sample.

Let us just have a closer look at the sample. In a first step the semiconductor heterostructure was fabricated by molecular beam epitaxy (MBE). In the next step the shape of the 2-DEG, including the confining arcs, are etched from the semiconductor. Finally the contact pads at both ends of the conductor and the gate are evaporated. See figure 8 for an illustration of the production process. The sample wafer containing many samples is then cut into pieces and one sample is glued onto a chip carrier and is electrically connected. Figure 6 shows a scanning electron microscope image of the conductor.

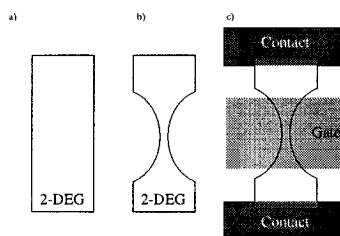


Figure 8: Illustration of the different fabrication steps: a) The semiconductor is fabricated and etched into a rectangular shape, b) The semiconductor is further etched into a constriction, approx 250 nm wide, c) the contact pads and the gate are evaporated

4. The Lab

Please be careful with the sample! These samples take quite a bit of time and effort to fabricate. Besides, the sample you are using have been fabricated else where, and thus we do not have access to direct replacements should the sample break!

Notice that static discharges can also destroy the sample. Do not touch the contacts!

- First connect the measurement set-up. Follow the illustration and the schematics given in figure 9 and 10.

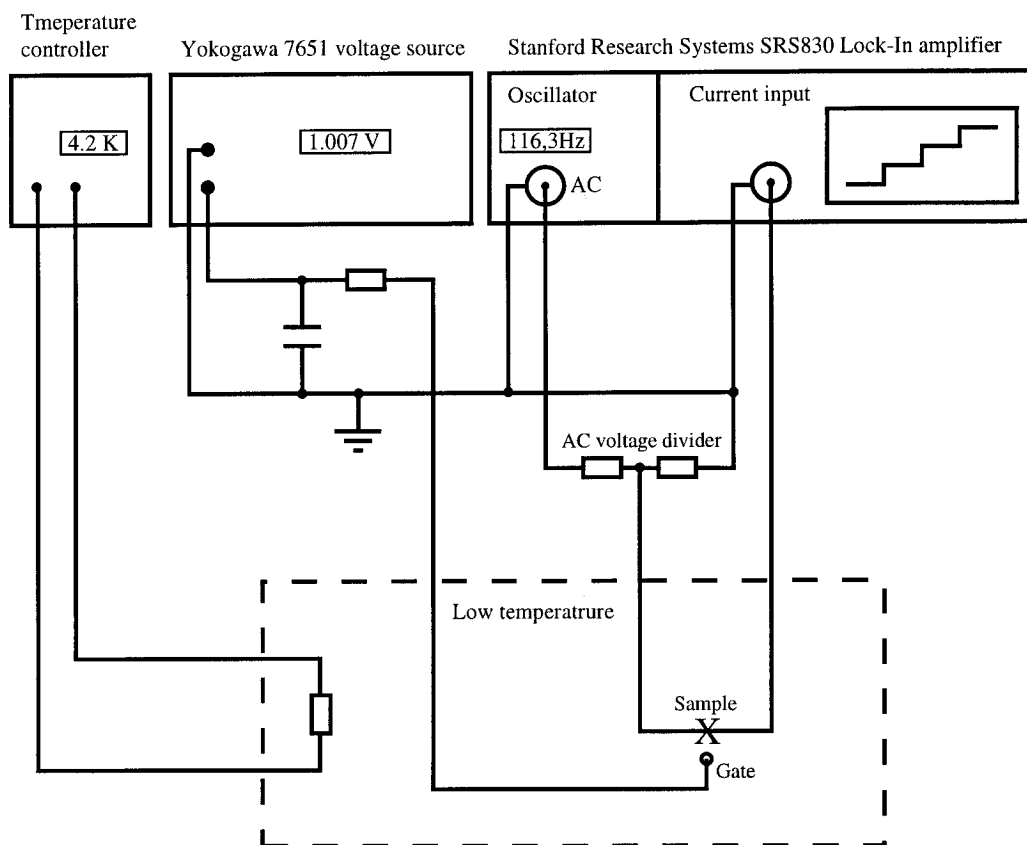


Figure 9: The measurement set-up.

- Mount the sample in the dipstick. Make sure that you have grounded the connections to the sample to avoid destroying the sample.
- Since the dipstick is closed we have to pump it and fill it with helium gas that will transport the heat between the sample and the liquid helium bath.
- Slowly cool down the sample.
- Measure: You should measure the conductance of the sample as a function of gate voltage at different temperatures.

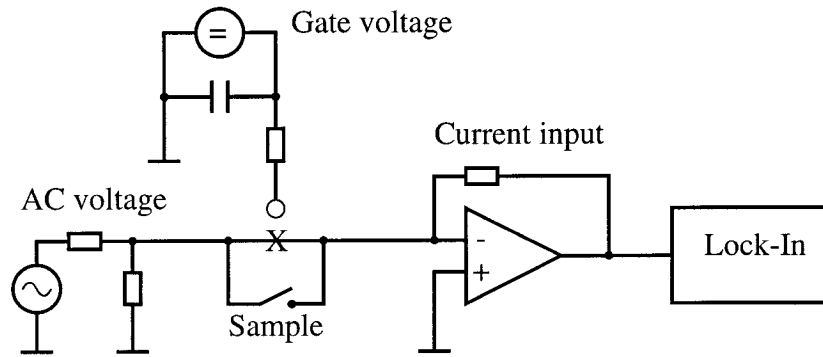


Figure 10: Schematic drawing of the measurement set-up.

After the lab you have to prepare a written report, two by two. It should have about five pages, detailing what you have done during the lab. It is important that one can follow what you have done experimentally. Finally you should analyze the data you have measured. Does it coincide with what we could expect from theory?

Hint: We will not only measure the resistance of the constriction, but we will also measure a serial resistance. You will have to find the value for this resistance and correct your data accordingly.

5. Acknowledgments

We would like to thank Poul Erik Lindelof, Anders Kristensen, and Carl Johan Marckmann at the Niels Bohr Institut in Copenhagen for providing us with the samples used in this experiment.

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A. Your opinion

Lab Me1, Kvantfysik F3

- | | | | | |
|---|---|-----|-------|--------|
| 1 | Följer Du föreläsningarna i kvantfysik? | Ja | Nej | |
| 2 | Hade Du läst igenom Lab-PM när Du kom hit? | Ja | Nej | |
| 3 | Läste Du motsvarande avsnitt i kursboken? | Ja | Nej | |
| 4 | Hur var Lab-PM? | Bra | Medel | Dåligt |
| 5 | Hur var den laborative delen? | Bra | Medel | Dåligt |
| 6 | Hur var handledaren? | Bra | Medel | Dåligt |
| 7 | Tycker Du laborationen belyser ett väsentligt avsnitt i kvantfysik? | Ja | Nej | |
| 8 | Övriga synpunkter: | | | |