Optimization of opportunistic maintenance for multi-component systems using stochastic programming

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A motivating example

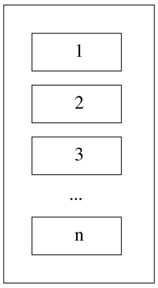
- Wind power turbine with 5 major components
- Crane is necessary for replacement of failed components
- Given failure of one component (opportunity), decide if other components should be replaced
- The decision is based on:
  - Components' life distributions (data)
  - Price of new component and cost of failure
  - Remaining life of the turbine

Opportunistic maintenance

- $x \in X$ - maintenance decision today
- $f : X \to \mathbb{R}$ cost of the maintenance today
- $Q_T : X \to \mathbb{R}$ expected value of future maintenance costs
- Optimization of opportunistic maintenance is to

$$\underset{x \in X}{\text{minimize}}\, f(x) + Q_T(x)$$

Multi-component problem considered



- $x_i = \begin{cases} 1 & \text{replace component } i \\ 0 & \text{otherwise} \end{cases} \quad i = 1, \dots, n$
- $f(x) := \sum_{i=1}^{n} c_i x_i$
- c_i replacement cost for component i
- Let r_i be # replacements of component i until time T
- Let s be # maintenance occasions until time T
- $Q_T(x) = E[\sum_{i=1}^{n} r_i c_i + sd]$
- d maintenance occasion cost

Two stage approach

- Let $\omega \in \Omega$ represent the outcomes of component lives
- Let $F(x, \omega)$ be the optimal maintenance schedule given the outcome ω
- Two stage problem

$$\underset{x \in X}{\text{minimize}}\, f(x) + E[F(x, \omega)]$$

Monte Carlo Approximation

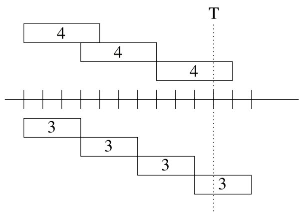
- Draw scenario ω_s from the distributions of lives, $s = 1, \dots, M$.
- Solve

$$\underset{x \in X}{\text{minimize}}\, f(x) + \sum_{s=1}^{M} F(x, \omega_s)/M$$

- Either one big mixed integer program or enumeration on x.

Evaluating $F(x, \omega)$ - the one scenario problem

If component lives are deterministic.



solved by model introduced by Dickman, Epstein and Wilamowsky [1].

The one scenario model

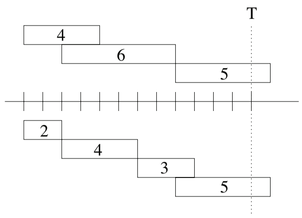
The objective function

$$\text{minimize} \sum_i \left(c_i x_{i0}^1 + \sum_{t,r} c_i (x_{it}^r - x_{it-1}^r) + \sum_t d z_t \right)$$

The constraints

$$\begin{aligned} x_{it}^r &\leq x_{it+1}^r \\ x_{it+1}^{r+1} &\leq x_{it}^r \\ x_{i0}^r &= 0, \quad r = 2, \dots \\ \sum_r x_{it}^r - x_{it-1}^r &\leq z_t \\ x_{it}^r &\leq x_{it+T_{ir+1}}^{r+1} \end{aligned}$$

Evaluating $F(x, \omega)$ - the one scenario problem

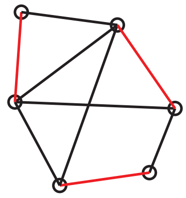


Definition

Given lives T_{ir} for every individual r of every component i, costs c_i, d and time horizon T, minimize the maintenance cost.

Properties

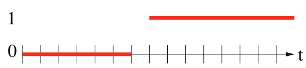
- The problem is NP-hard (reduction from vertex cover).



- We can relax the integrality on x_{it}^r.
- All inequalities are facet defining.

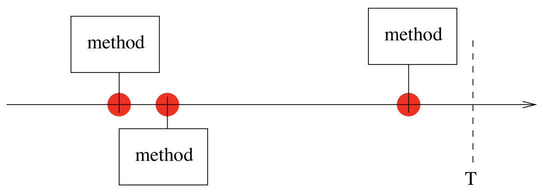
The variables

$$x_{it}^r = \begin{cases} 1 & \text{individual } r \text{ of component } i \\ & \text{is/has been replaced at time } t, \\ 0 & \text{otherwise.} \end{cases}$$



$$z_t = \begin{cases} 1 & \text{maintenance performed at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Simulation



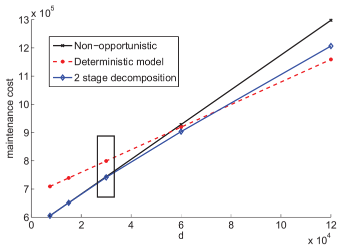
- 100 runs for each method
- Methods compared:
  - Non-opportunistic maintenance
  - Deterministic model
  - Stochastic 2 stage model

Test case

- Data from Poore and Walford NREL [2]
- Crane mobilization cost $ 30 000 (is varied)

component	qt	α [years]	β	c [k$]
blades – st. rep.	3	0.0025^{-1}	1	100
blades – non st. rep.	1	0.05^{-1}	1	48
pitch bearing	3	0.0025^{-1}	1	43
main bearing	1	0.0025^{-1}	1	91
gearbox – g. & brgs	1	0.0025^{-1}	1	344
gearbox – brgs, all	1	20	3.5	222
gearbox – high sp.	1	20	3.5	137
generator – rot. & brgs	1	0.0025^{-1}	1	132
generator – brgs.	2	17	3.5	36

Results



Results

d	method	abs. impr.	rel. impr.
30 000	det. model	-55 000	-7.4 %
30 000	stoch. model	2 500	0.3 %
60 000	det. model	9 200	1.0 %
60 000	stoch. model	25 200	2.7 %
120 000	det. model	138 000	10.7 %
120 000	stoch. model	91 500	7.0 %

Conclusions and future research

Conclusions

- Optimization of opportunistic maintenance reduces costs
- The stochastic model always performs better than the non-opportunistic approach
- Only small problems can be solved

Future research

- An efficient method to solve the two stage problem
- The multistage problem
- More complex models incorporating production planning and/or whole windfarms

References

B. Dickman, S. Epstein and Y. Wilamowsky: *A mixed integer linear programming formulation for multi-component deterministic opportunistic replacement*, The Journal of the Operational Research Society of India, 28 (1991), pp. 165-175

R. Poore and C. Walford: *Development of an Operations and Maintenance Cost Model to Identify Cost and Energy Savings for Low Wind Speed Turbines*, Report National Renewable Energy Laboratory, January 2008