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Title: The Complexity of Translationally Invariant Problems beyond Ground State Energies

Abstract:
The physically motivated quantum generalization of k-SAT, known as the k-Local Hamiltonian (k-LH) problem, is well-known to be QMA-complete. What is surprising, however, is that while the former is easy on 1D Boolean formulae, the latter remains hard on 1D local Hamiltonians, even if all constraints are identical [Gottesman, Irani, FOCS 2009]. Such "translation-invariant" systems are much closer in structure to what one might see in Nature. Moving beyond LH, a key physical problem is that of computing properties of the ground space (i.e. "solution space") itself. In this work, we focus on two such recent problems: Simulating local measurements on the ground space (APX-SIM, analogous to computing properties of optimal solutions to MAX-SAT formulae) [Ambainis, CCC 2014], and deciding if the low energy space has an energy barrier (GSCON, analogous to classical reconfiguration problems) [G, Sikora, ICALP 2015]. These problems are known to be P^QMA[log]- and QCMA-complete, respectively, in the general case. Yet, to date, it is not known whether they remain hard in such "simple" 1D translationally invariant systems.

In this work, we show that the 1D translationally invariant versions of both APX-SIM and GSCON are intractable, namely are P^QMA_EXP- and QCMA_EXP-complete, respectively. Each of these results is attained by giving a respective generic "lifting theorem". For example, for APX-SIM we give a framework for "lifting" any abstract local circuit-to-Hamiltonian mapping H (satisfying mild assumptions) to hardness of APX-SIM on the family of Hamiltonians produced by H, while preserving the structural and geometric properties of H (e.g. translation invariance, geometry, locality, etc). Each result also leverages counterintuitive properties of our constructions: for APX-SIM, we "compress" the answers to polynomially many parallel queries to a QMA oracle into a single qubit. For GSCON, we show soundness even against adversaries which act on all but a single qudit in the system in any time step.

Based on joint work with Johannes Bausch (Cambridge) and James Watson (U College London). Preprint: https://arxiv.org/pdf/2012.12717.pdf.