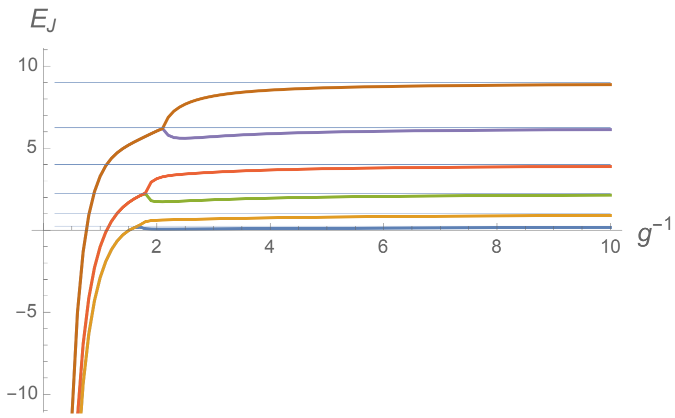
Master project: Bethe ansatz solution for interacting fermions

Background: The many-body problem in physics describes a system of many particles (for example, electrons in a solid or atoms in a gas) that interact with a pairwise potential (for example, the Coulomb interaction between electrons or a short-range repulsive potential between two atoms). Most methods in theoretical physics that compute the properties of a many-body system rely on a small interaction expansion parameter, often uncontrolled approximations, or large-scale numerical computation, and fail at strong interactions. A very important exception is the Bethe ansatz, which is a technique from mathematical physics that provides an exact algebraic solution of certain models in one space dimension. The method is exact, which means that it provides a solution of the many-body problem even at strong interactions, where most other techniques fail.

The Bethe ansatz has gained increased interest in recent years since a number of experiments are described by models that are solved by the Bethe ansatz, for example, quantum gases in a one-dimensional trap, semiconductor quantum dots, or small superconductors. This means that on the one hand, the Bethe ansatz may be confirmed in experiments, and on the other hand, new experimental protocols inspire new applications and variations of this method.



Left: Sketch of an energy spectrum for twelve interacting fermions (spin-up and spin-down) in a box obtained using the Bethe ansatz. Plotted are energy levels of fermion pairs as a function of the interaction strength g. The thin lines show the non-interacting energy levels for comparison. Right: The QR code links to a feature article in Physics Today called "The Bethe ansatz after 75 years" written by M. T. Batchelor.

Project proposal: In this theory project, you will apply the Bethe ansatz to models of interacting fermions in one dimension that describe electrons in a nanoscale wire. You will derive the Bethe ansatz and then solve it for a small number of particles. The first part is analytical work, the second part requires a solution of a coupled system of equations, which is done numerically.

A good knowledge of quantum mechanics is essential, some knowledge of many-body theory would be useful. Knowledge of a computer algebra system or some programming language is helpful but can also be acquired during the project.

Office space at Physics is expected to be available, but depending on future corona-virus restrictions we might have to work remotely.

Supervisor:
Johannes Hofmann, johannes.hofmann@physics.gu.se, Department of Physics, GU.