PhD course in Nonlinear Optimization
Mathematical Sciences, Chalmers

2018-09-27

Michael Patriksson
Ann-Brith Strömberg

Key words

Convex sets and functions, separation, cones and polarity, polyhedral sets, extreme points and directions, subgradients, minima and maxima, support functions, the Fritz John and Karush-Kuhn-Tucker conditions, constraint qualifications, Farkas' Lemma, sensitivity analysis, Lagrangian duality, duality gap, saddle points, Lagrangian dual problem, nonsmooth optimization, decomposition, closed algorithmic maps, unconstrained optimization, line searches, gradient-based optimization methods, coordinate search, convergence rates, derivative-free optimization, penalty and barrier methods, interior point methods, feasible-direction methods.

Credits

12 hp in the PhD program: 7.5 hp for the theory part, 4.5 for the algorithmic part

Literature

- M S Bazaraa, H D Sherali and C M Shetty: *Nonlinear Programming: Theory and Algorithms* (Wiley, 2006) (main course book)
- D P Bertsekas: *Nonlinear Programming* (Athena, 1999) (supplementary material)
- A few selected articles (typically more recent) will also be included

Examination

Exercises, oral examination and a project.

Project

Implementation and evaluation of some iterative methods for nonlinear optimization.

Reading list by topic

Bazaraa, Sherali, & Shetty, Chapter 1: Introduction

- Exercises: 2, 6, 12, 13
- Topics for oral exam
    - Section 1.3

Bazaraa, Sherali, & Shetty, Chapter 2: Convex sets

- Exercises: 1−2, 9, 11, 16−21, 25, 27, 29−31, 38−39, 42, 44, 46, 52−54
- Topics for oral exam
    - Theorems 2.1.6, 2.3.1, 2.4.1, 2.4.4, 2.4.5, 2.5.4, 2.6.5, 2.6.7
- Additional topic
    - Generalization of Theorem 2.4.5 to nonlinear systems

Bazaraa, Sherali, & Shetty, Chapter 3: Convex functions and generalizations

- Exercises: 9−10, 12, 14, 16−18, 20−24, 26−27, 31−33, 38, 40, 43, 45−47, 50−52, 54−57, 69
- Topics for oral exam
    - Theorems 3.1.3, 3.4.2−3.4.4, 3.4.7
    - The subdifferential
- Additional topics
    - Generalized convex functions and monotone mappings

    - Generalizations of Theorem 3.4.4 (establish a similar statement for non-differentiable convex functions; re-establish Theorem 3.4.4 where f in C^1 replaces f in C^2)

Bertsekas, Chapter 2.1: Optimality conditions

- Exercises: 1.10−1.12

Bazaraa, Sherali, & Shetty, Chapter 4: The Fritz John and the Karush-Kuhn-Tucker optimality conditions

- Exercises: 4, 6, 9, 12−17, 19−20, 24, 26, 29−32, 34, 37, 39, 41, 45−48
- Topics for oral exam:
    - Theorems 4.2.2, 4.2.5, 4.2.8, 4.2.13, 4.2.15, 4.2.16
    - Differences between FJ and KKT
    - Example 4.4.4

Bertsekas, Chapter 3.2: Sufficient conditions and sensitivity analysis

- Exercises: 3.1−3.3
- Additional topic
    - Upper semicontinuity of solutions to linear programs

Bazaraa, Sherali, & Shetty, Chapter 5: Constraint qualifications

- Exercises: 3, 5−6, 12−16, 20
- Topics for oral exam:
    - Relations among the CQs (strengths), and their role in the convex and linear cases

Bazaraa, Sherali, & Shetty, Chapter 6: Lagrangian duality and saddle point optimality conditions

- Exercises: 1, 3−7, 9, 13−14, 16, 18−19, 22, 25−26, 29−30, 34, 36−38, 41−44
- Topics for oral exam
    - Geometry of the dual problem
    - Theorems 6.2.5, 6.3.1−6.3.3, 6.3.11, 6.5.1

Bertsekas, Chapter 5: Duality and convex programming *[Section 5.4 only for orientation]*

- Exercises: 3.1, 3.3
- Topics for oral exam
    - Duality gaps in integer optimization
    - Propositions 5.1.1, 5.1.4−5.1.6, 5.3.1

Bertsekas, Chapter 6: Dual methods *[see also Bazaraa, Sherali, & Shetty, Section 8.9]*

- Exercises: 2.1; 3.1, 3.4−3.5, 3.8, 3.10, 3.12
- Topics for oral exam
    - Propositions 6.1.2, 6.3.1
- Additional topics
    - Ergodic convergence [Shor, Larsson et al.]
    - Convexification through dualization

Bazaraa, Sherali, & Shetty, Chapter 7: The concept of an algorithm

- Exercises: TBA
- Topics for oral exam
    - Closedness of a mapping
- Additional topics
    - Closedness versus upper semicontinuity
    - Spacer steps [Luenberger]

Bazaraa, Sherali, & Shetty, Chapter 8: Unconstrained optimization

- Exercises: TBA
- Topics for oral exam
    - Relationships between gradient and trust region methods
    - Theorem 8.8.3

Bertsekas, Chapter 1: Unconstrained optimization *[Section 1.9 only for orientation]*

- Exercises: 2.4, 2.5−2.6, 2.10
- Additional topics:
    - The Wolfe condition in line search methods [Dennis and Schnabel, 1983, 6.3]
    - Convergence of steepest descent methods [Bertsekas and Tsitsiklis, SIAM J Optim, 2000]
    - Convergence of Gauss-Seidel and Jacobi methods [Bertsekas and Tsitsiklis, 1989, 5.1, 5.5−5.6]
    - Pattern search [Lewis, Torczon, and Trosset, OPTIMA, 1998]
    - Trust region methods [Dennis and Schnabel, 1983, 6.4]
    - Non-monotone and curvilinear searches [Toint, SIAM J Sci Comput, 1996; Ferris, Lucidi, and Roma, COAP, 1996]
    - Closedness of some iterative methods [Patriksson]

Bazaraa, Sherali, & Shetty, Chapter 9: Penalty and barrier functions

- Exercises: TBA
- Topics for oral exam
    - Interpretations
    - Relationships

Bertsekas, Chapter 2: Optimization over a convex set

- Exercises: 2.2−2.3, 2.6−2.7, 3.2, 3.6, 3.8, 7.2
- Topics for oral exam
    - Relationships between optimality and the construction of improving directions

Bazaraa, Sherali, & Shetty, Chapter 10: Methods of feasible directions *[for orientation]*

- Exercises: TBA
- Topic for oral exam
    - Relationships between the optimality conditions and the basis for the construction of improving directions
- Additional topics
    - Simplicial decomposition [Hearn et al., Patriksson]