Optimization approach in the design of approximate cloaking structures

Bachelor Thesis project

Abstract

In this project we will consider the problem of the construction of the approximate cloaking structure of arbitrary geometry. We reformulate this problem as an optimization problem for the Tikhonov functional which is minimized on adaptively locally refined meshes. These meshes are refined only in places where the nanophotonic structure should be designed. Our special symmetric mesh refinement procedure allows the construction of different nanophotonic structures.

The existing C++/PETSc [22] software package WavES (waves24.com) will be used for computations. The part of the project will be further development of this software for this special application. This will be continuation of previous work which is done in [14] and with collaboration with Chalmers Area of Advance Nanoscience and Nanotechnology.

1 Introduction

The goal of this work is to develop a new optimization algorithm that can construct arbitrary nanophotonic structures from desired scattering parameters. Nanophotonics is the study of the interaction of electromagnetic waves with structures that have feature sizes equal or smaller than the wavelength of the waves. Examples are photonic crystals (structured on the wavelength scale), metamaterials (subwavelength structured media with new optical properties that are not available from natural materials) and plasmonic devices (exploiting collective excitations in metals that result in strong field enhancement) [19, 21, 23, 28].

We should develop a nonparametric optimization algorithm that can find inner structure of the domain with arbitrary geometry. To do that we apply an adaptive finite element method of [2, 7] with iterative choice of the regularization parameter [1].

2 Statement of the forward and inverse problems

Let $x = (x_1, x_2)$ denotes a point in $\mathbb{R}^2$ in an unbounded domain D. In this work we consider the propagation of electromagnetic waves in two dimensions with a field polarization. Thus, we model the wave propagation by the following Cauchy problem for the scalar wave equation:

$$\begin{cases} \varepsilon(x)\frac{\partial^2 E}{\partial t^2} - \triangle E = \delta(x_2 - x_0)p(t) & \text{in } \mathbb{R}^2 \times (0,\infty), \\ E(x,0) = f_0(x), \quad E_t(x,0) = 0 & \text{in } D. \end{cases} \tag{1}$$

Here, E is the electric field generated by the plane wave $p(t)$ which is incident at $x_2 = x_0$ and propagates along x_2 axis, $\varepsilon(x)$ is the spatially distributed dielectric permittivity. We note that in this work we use the single equation (1) instead of the full Maxwell's equations, since in [4] was demonstrated numerically that in the similar numerical setting, as we will use in this note,

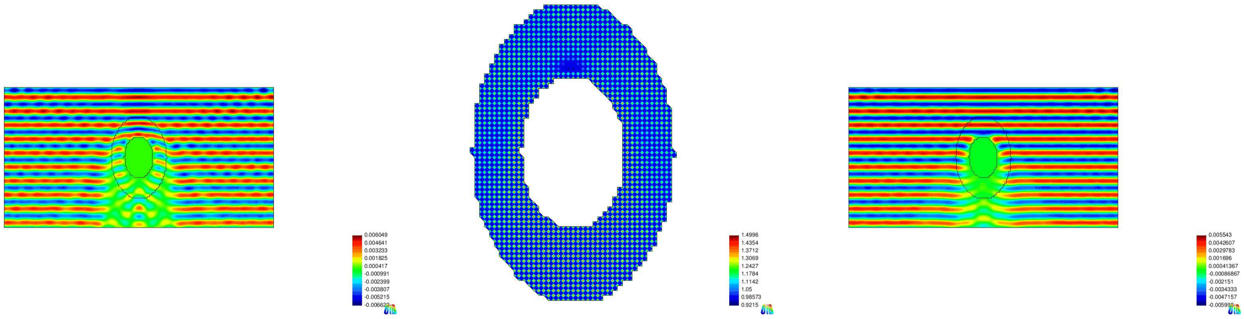

Figure 1: *Designing new nano-materials with prescribed properties. Goal: construct a such physically reliable material (can be constructed in reality) which will give a small reflections as possible. Left: reflections from initial known material, middle: constructed new optimized material, right: reflections from optimized new material.*

other components of the electric field are negligible compared to the initialized one. We also note that a scalar model of the wave equation was used successfully to validate reconstruction of the dielectric permittivity function with transmitted [10, 11] and backscattered experimental data [12, 13, 20, 24, 25].

Let now $D \subset \mathbb{R}^2$ be a convex bounded domain with the boundary $\partial D \in C^2$. We denote by $D_T := D \times (0,T), \partial D_T := \partial D \times (0,T), T > 0$ and assume that

$$f_0 \in H^1(D), \varepsilon(x) \in C^2(D). \tag{2}$$

For computational solution of (1) we use the domain decomposition finite element/finite difference (FE/FD) method of [3, 5] which was applied for the solution of different coefficient inverse problems for the acoustic wave equation in [2, 3, 7]. To apply method of [3, 5] we decompose D into two regions D_{FEM} and D_{FDM} such that the whole domain $D = D_{FEM} \cup D_{FDM}$, and $D_{FEM} \cap D_{FDM} = \emptyset$. In D_{FEM} we use the finite element method (FEM), and in D_{FDM} we will use the Finite Difference Method (FDM). We avoid instabilities at interfaces between FE and FD domains since FE and FD discretization schemes coincide on two common structured layers with $\varepsilon(x) = 1$ in them.

Let the boundary ∂D be such that $\partial D = \partial_1 D \cup \partial_2 D \cup \partial_3 D$ where $\partial_1 D$ and $\partial_2 D$ are, respectively, front and back sides of the domain D, and $\partial_3 D$ is the union of left, right, top and bottom sides of this domain. At $S_{T_1} := \partial_1 D \times (0,T)$ and $S_{T_2} := \partial_2 D \times (0,T)$ we have time-dependent backscattering and transmission observations, correspondingly. We define $S_{1,1} := \partial_1 D \times (0, t_1]$, $S_{1,2} := \partial_1 D \times (t_1, T)$, and $S_3 := \partial_3 D \times (0,T)$. We also introduce the following spaces of real valued functions

$$\begin{aligned} H_E^1(D_T) &:= \{w \in H^1(D_T) : w(\cdot, 0) = 0\}, \\ H_\lambda^1(D_T) &:= \{w \in H^1(D_T) : w(\cdot, T) = 0\}, \\ U^1 &= H_E^1(D_T) \times H_\lambda^1(D_T) \times C\left(\overline{D}\right), \end{aligned} \tag{3}$$

and define standard L_2 inner product and space-time norms, correspondingly, as

$$\begin{aligned} ((u,v))_{D_T} &= \int_D \int_0^T uv \; dxdt, \; \|u\|^2_{L_2(D_T)} = ((u,u))_{D_T}, \\ (u,v)_D &= \int_D uv \; dx, \; \|u\|^2_{L_2(D)} = (u,u)_D. \end{aligned}$$

In our computations we have used the following model problem

$$
\begin{cases}
\varepsilon \frac{\partial^2 E}{\partial t^2} - \triangle E = 0 & \text{in } D_T, \\
E(x,0) = f_0(x), \quad E_t(x,0) = 0 & \text{in } D, \\
\partial_n E = p\,(t) & \text{on } S_{1,1}, \\
\partial_n E = -\partial_t E & \text{on } S_{1,2}, \\
\partial_n E = -\partial_t E & \text{on } S_{T_2}, \\
\partial_n E = 0 & \text{on } S_3.
\end{cases} \tag{4}
$$

In (1) we use the first order absorbing boundary conditions [17]. These conditions are exact since we initialize the plane wave orthogonal to the domain of propagation.

We choose the coefficient $\varepsilon(x)$ in (1) such that

$$
\begin{cases}
\varepsilon\,(x) \in (0, M]\,, M = const. > 0, & \text{for } x \in D_{FEM}, \\
\varepsilon(x) = 1 & \text{for } x \in D_{FDM}.
\end{cases} \tag{5}
$$

We consider the following inverse problem

Inverse Problem (IP)

Let the coefficient $\varepsilon\,(x)$ in the problem (4) satisfy conditions (5) and assume that $\varepsilon\,(x)$ is unknown in the domain $D \diagdown D_{FDM}$. Determine the function $\varepsilon\,(x)$ in (4) for $x \in D \diagdown D_{FDM}$, assuming that the following function $\widetilde{E}\,(x,t)$ is known

$$
E\,(x,t) = \widetilde{E}\,(x,t)\,,\ \forall\,(x,t) \in S_{T_1} \cup S_{T_2}. \tag{6}
$$

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