Noncommutativity in the North

A Markus Wallenberg symposium

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Chalmers/University of Gothenburg 14-18 March 2022

Abstracts

Arkadiusz Bochniak (Jagiellonian University)
Quantum spaces of maps for quantum correlations

For given quantum spaces, we study the C*-algebra corresponding to the quantum space of maps between them. We define a universal operator system inside this C*-algebra and discuss its properties, e.g. the hyperrigidity of its embedding. This construction is then applied to study classes of quantum correlations, including the notion of synchronicity. This talk is based on joint work with P. Kasprzak and P. M. Sołtan.

Kevin Aguyar Brix (University of Glasgow)
Ideal structure of C^*-algebras of commuting local homeomorphisms

I will report on on-going work with Toke Carlsen and Aidan Sims on a dynamical identification of the complete ideal lattice of C*-algebras built from commuting local homeomorphism. Examples of this class include graph C*-algebras, their topological analogues, higher rank graphs as well as certain multidimensional subshifts. Our results rely on the existence of certain well behaved families of bisections.

Rui Dong (Radboud University)
The gauge group and perturbation semigroup of an operator system

The perturbation semigroup was first defined in the case of $*-$algebras by Chamseddine, Connes and van Suijlekom. In this paper we take $\mathcal{E}$ as a concrete operator system with unit. We first give a definition of gauge group $\mathcal{G}(\mathcal{E})$ of $\mathcal{E}$, after that we give the definition of perturbation semigroup of $\mathcal{E}$, and the closed perturbation semigroup of $\mathcal{E}$ with respect to the Haagerup tensor norm. We also show that there is a continuous semigroup homomorphism from the closed perturbation semigroup to the collection of unital completely bounded Hermitian maps over $\mathcal{E}$. Finally we compute the gauge group and perturbation semigroup of the Toeplitz system as an example.

Eske Ewert (Leibniz University Hannover)
Pseudodifferential operators as generalized fixed points

Generalized fixed point algebras were introduced by Rieffel to define a noncommutative analogue of proper group actions on spaces. In this talk, I will explain how different C^*-algebras of pseudodifferential operators arise as generalized fixed point algebras of $\mathbb{R}_{>0}$-actions. In particular, pseudodifferential calculi on filtered manifolds and graded Lie groups can be obtained in this way. By computing the spectrum of the respective symbol algebras we recover Fredholm conditions like the Rockland condition.

Marzieh Forough (Czech Technical University)
C^*-algebras associated to homeomorphisms twisted by vector bundles over finite dimensional spaces

In this talk, I will discuss the structure of Cuntz-Pimsner algebras associated to C*-correspondences which as right modules have the structure of a module of sections of a vector bundle over a compact metric space X and whose left action is given by a homeomorphism on X. I will explain when the corresponding Cuntz-Pimsner algebra is simple and determine its trace space. When the C*-correspondence is coming from an aperiodic homeomorphism, I will discuss that the associated Cuntz-Pimsner algebra has finite nuclear dimension. This leads us to a criterion determining when the resulting C*-algebra has stable rank one or is purely infinite. Moreover, I will briefly explain the twisted groupoid picture of this class of C*-algebras.
This talk is based on joint work with Marai Stella Adamo, Dawn Archey, Magdalena Georgescu, Ja A Jeong, Karen Strung and Maria Grazia Viola and an ongoing work with Maria Stella Adamo and Karen Strung.

James Gabe (University of Southern Denmark)
The dynamical Kirchberg-Phillips theorem

In celebrated work, Kirchberg and Phillips classified all separable, nuclear, simple, purely infinite C*-algebras (aka Kirchberg algebras) using KK-theory. I will talk about a generalization of this theorem to the C*-dynamical setting. For any second countable locally compact group G, we show that any Kirchberg algebra with an amenable and suitably outer G-action is classified up to cocycle conjugacy using equivariant KK-theory. This is joint work with Gábor Szabó.

Erik Habbestad (University of Oslo)
Subproduct systems with quantum group symmetry

We introduce a class of subproduct systems of finite dimensional Hilbert spaces whose fibers are defined by the Jones-Wenzl projections in Temperley-Lieb algebras. The quantum symmetries of a subclass of these systems are the free orthogonal quantum groups. Using this we are able to describe the corresponding Toeplitz and Cuntz-Pimsner algebras.

Jens Kaad (University of Southern Denmark)
Quantum Gromov-Hausdorff continuity of quantum $\mathrm{SU}(2)$

In this talk we investigate the spectral metric properties of quantum $\mathrm{SU}(2)$. These spectral metric properties are encoded by a pair of twisted derivations coming from the pairing between the quantum enveloping algebra and the coordinate algebra for quantum $\mathrm{SU}(2)$. We shall see that these twisted derivations provide quantum $\mathrm{SU}(2)$ with the structure of a compact quantum metric space (in the sense of Rieffel). Moreover, these compact quantum metric spaces vary continuously in the q-deformation parameter with respect to the quantum Gromov-Hausdorff distance. This includes the special value $q = 1$, where we recover the classical 3-sphere equipped with the round metric. This talk can be viewed as a continuation of the talk given by David Kyed in so far that the convergence results obtained for quantum $\mathrm{SU}(2)$ rely on the convergence results for the Podleś sphere. The talk is based on joint work with David Kyed.

Matthew Kennedy (University of Waterloo)
The ideal intersection property for essential groupoid C^**-algebras*

I will discuss recent work characterizing the ideal intersection property for essential C^*-algebras of étale groupoids with locally compact Hausdorff space of units. For Hausdorff groupoids, this C^*-algebra coincides with the reduced C^*-algebra. In the minimal case, the ideal intersection property is equivalent to simplicity, so as a consequence we obtain a characterization of étale groupoids that are C^*-simple.
This is joint work with Kim, Li, Raum and Ursu.

Julian Kranz (University of Münster)
Classifiability of crossed products by nonamenable groups

In recent joint work with E. Gardella, S. Geffen and P. Naryshkin, we exhibit a large class of nonamenable groups with the property that any amenable minimal and topologically free action on a compact space gives rise to a purely infinite crossed product. As a consequence, such crossed products are classified by K-theory. Our class of groups contains many groups acting on nonnegatively curved spaces. I will explain how we use the geometry of the associated boundary actions to obtain a combinatorial property of the group that we call paradoxical towers. This combinatorial property can then be used to establish dynamical comparison for any amenable minimal action of the group on a compact space, which in turn implies pure infiniteness of the crossed product under the additional assumption of topological freeness.

David Kyed (University of Southern Denmark)
The Podles spheres converge to the sphere

The Podleś spheres, which are q-deformed analogues of the 2-sphere, are among the most classical objects in non-commutative geometry, but only quite recently their structure as quantum metric spaces has begun to unravel. In my talk, I will first provide a basic introduction to Rieffel's theory of compact quantum metric spaces and his non-commutative counterpart to the Gromov-Hausdorff distance, and then present some recent results showing that the quantised 2-spheres converge (in the quantum Gromov-Hausdorff distance) to the classical round 2-sphere as the deformation parameter q tends to 1. The talk is based on joint work with Konrad Aguilar, Thomas Gotfredsen and Jens Kaad, and constitutes the first of two talks on the topic. The second talk, given by Jens Kaad, will describe how one can further utilise these results to obtain new insights into the non-commutative metric geometry of quantum $\mathrm{SU}(2)$.

Franz Luef (NTNU Trondheim)
Approximating Heisenberg modules over irrational noncommutative tori by finite-dimensional ones

We show that vector bundles over an irrational noncommutative torus may be approximated by vector bundles over finite-dimensional matrix algebras that converge to the irrational noncommutative torus with respect to the module norm of the generators, when the matrix algebras converge in the quantum Gromov-Hausdorff distance to the irrational noncommutative torus.
The argument is based on the procedure of restriction and periodization of the generators of the Heisenberg module which is well-established in the theory of Gabor frames. The latter is of relevance since generators of Heisenberg modules are atoms for Gabor frames. We will make ample use that the generators of Heisenberg modules might be chosen from Feichtinger's algebra, which provides a convenient setting for the study of Heisenberg modules.
This is joint work with Mads S. Jakobsen.

Diego Martinez (University of Münster)
Exactness and geometric properties of inverse semigroups

Inverse semigroups, and groupoids, are closely related, and both generalize the notion of a group in different ways. In this talk, we will explore when an inverse semigroup is exact, in the sense that its reduced C^*-algebra is exact. To this end, we will equip the inverse semigroup with a (suitably) proper and right invariant metric, and prove the resulting metric space has Yu's property A precisely when the semigroup is exact. Time permitting, we will also characterize when these semigroups have asymptotic dimension 0, giving a subclass of the former one.

Magdalena Musat (University of Copenhagen)
Convex structure of unital quantum channels, factorizability and traces on the universal free product of matrix algebras

Factorizable quantum channels, introduced by C. Anantharaman-Delaroche within the framework of operator algebras, have recently found important applications in the analysis of quantum information theory, revealing new infinite dimensional phenomena, and leading to reformulations of the Connes Embedding Problem. In earlier work with U. Haagerup, we showed that there are non-factorizable channels in all dimensions larger than or equal to 3, and that each is a counterexample to a conjectured restoration in the asymptotic limit of the classical Birkhoff theorem. Recently completed work, joint additionally with M.B. Ruskai, provides a systematic recipe for constructing large classes of non-factorizable maps, through further analysis of the convex structure of unital quantum channels. In the second part of the talk, I will discuss a new viewpoint, joint with M. Rørdam, leading to central questions in C^*-algebra theory.

Petr Naryshkin (University of Münster)
Almost finiteness and classifiability of crossed products

It has been a long-standing problem in the classification theory of C*-algebras to find sufficient conditions for a topological dynamical system to give rise to a classifiable crossed product. To address this problem, in 2015 Kerr introduced the notion of almost finiteness and showed that it implies $\mathcal{Z}$-stability of the crossed product which is now known to be the key regularity property in classification. In this talk, we will present some recent results showing that certain classes of systems are almost finite.

Sanaz Pooya (Stockholm University)
The Baum-Connes assembly map for certain subgroups of $\mathbb{Z}^2 \rtimes \mathrm{GL}(2,\mathbb{Z})$

The Baum-Connes conjecture for a group G predicts that a specific map from equivariant K-homology to K-theory of natural objects constructed from G is an isomorphism. This specific map is natural in G and called the assembly map. While the conjecture is proved for large classes of groups, it is still open for linear groups in general. In this talk we discuss an alternative and explicit method of proving the Baum-Connes conjecture for some semi-direct products $G = \mathbb{Z}^2 \rtimes \Gamma$, where Γ is a non-amenable subgroup of $\mathrm{GL}(2,\mathbb{Z})$. This is feasible thanks to the presence of a 3-dimensional model for the classifying space for proper actions of G combined with detailed understanding of the K-theory of C*-algebras of wallpaper groups. Besides providing a hands-on proof for the Baum-Connes conjecture in these cases, our method elucidates the assembly map for groups we study. This is joint work with R. Flores, A. Valette and A. Zumbrunnen.

Mikael Rørdam (University of Copenhagen)
Traces and quasi-traces on C-algebras*

We give an overview of older results about the existence of traces, respectively, quasi-traces on C*-algebras, and Haagerup's proof that quasi-traces on exact C*-algebras are traces (in the version of Haagerup-Thorbjoersen). We discuss stability properties of obstructions to having traces, respectively, quasi-traces, and use this to describe when ultra-powers of a sequence of C*-algebras admit a (quasi-)trace. We give an example of an ultra-power of a sequence of simple unital C*-algebras neither of which admit a quasi-trace, but where the ultra-power does. This example illustrates a theorem of Ozawa describing traces on ultra-power of C*-algebras. We have not yet been able to decide if the ultra-power in our construction in fact admits a trace. This is a join work with my PhD student Henning Milhøj.

Adam Skalski (IMPAN Warszawa)
Quantum Hecke algebras

A Hecke pair (Γ, Λ) is a group Γ with an almost normal subgroup Λ. To such a pair one can associate a natural *-algebra faithfully represented on $\ell^2(\Gamma/\Lambda)$, the Hecke algebra of (Γ, Λ), whose operator algebraic completions exhibit interesting phenomena related to the group/representation theoretic properties of Γ and Λ. I will present the analogous construction in the setting of discrete quantum groups, explaining the natural extension of the 'almost normal' condition, and focusing on the new features appearing only in the quantum world. In particular I will describe a quantum version of the Schlichting completion, which associates to a quantum Hecke pair (Γ, Λ) a new pair (G, H), where G is a locally compact quantum group with a compact open quantum subgroup H, in such a way that the corresponding Hecke algebra does not change.
Based on joint work with Roland Vergnioux and Christian Voigt.

Tatiana Shulman (Chalmers/University of Gothenburg)
Central sequence algebras via nilpotent elements

Central sequence algebras play an important role in C*-algebra and von Neumann algebra theory. A central sequence in a C*-algebra is a sequence (x_n) of elements such that $[x_n, a]$ converges to zero, for any element a of the C*-algebra. In von Neumann algebra setting one typically means the convergence with respect to tracial norms, while in C*-theory it is with respect to the C*-norm.
In this talk we will consider the C*-theory version of central sequences. We will discuss properties of central sequence algebras and in particular address a question of J. Phillips and of Ando and Kirchberg of which separable C*-algebras have abelian central sequence algebras. Joint work with Dominic Enders.

Karen Strung (Czech Academy of Sciences)
On the classification of crossed products of commutative C-algebras by Hilbert bimodules*

Building on some of the ideas in Marzieh Forough's talk, I will talk about crossed products of commutative C*-algebras by Hilbert bimodules, a special case of the Cuntz–Pimsner construction. When the Hilbert bimodule comes from a homeomorphism of mean dimension zero twisted by a line bundle, the resulting C*-algebra absorbs the Jiang–Su algebra. With no assumptions on the mean dimension, the tensor product of two or more such C*-algebras also absorbs the Jiang–Su algebra. This entails their classification by the Elliott Invariant. This is joint work with Ja A Jeong and Marzieh Forough.

Hannes Thiel (University of Kiel)
Are C-algebras determined by their linear and orthogonality structure?*

It is well-known that every C*-algebra is determined by its linear and multiplicative structure: Two C*-algebras are *-isomorphic if and only if they admit a multiplicative, linear bijection.
We study if instead of the whole multiplicative structure it suffices to record when two elements have zero product. While it is not clear if every C*-algebra is determined this way, we obtain many positive results. In particular, two unital, simple C*-algebras are *-isomorphic if and only if they admit a linear bijection that preserves zero products. This is joint work with Eusebio Gardella.

Walter Van Suijlekom (Radboud University)
Noncommutative spaces at finite resolution

We extend the traditional framework of noncommutative geometry in order to deal with two types of approximation of metric spaces. On the one hand, we consider spectral truncations of geometric spaces, while on the other hand, we consider metric spaces up to a finite resolution. In our approach the traditional role played by C*-algebras is taken over by so-called operator systems. Essentially, this is the minimal structure required on a space of operators to be able to speak of positive elements, states, pure states, etc. We consider C*-envelopes and introduce a propagation number for operator systems, which we show to be an invariant under stable equivalence and use it to compare approximations of the same space. We illustrate our methods for concrete examples obtained by spectral truncations of the circle, and of metric spaces up to finite resolution. The first are operator systems of finite-dimensional Toeplitz matrices, the second are suitable subspaces of the compact operators. We also analyze the cones of positive elements and the pure state spaces for these operator systems, which turn out to possess a very rich structure.
(based on joint work with Alain Connes)

Hang Wang (East China Normal University)
Topological K-theory for discrete groups and index formula

Topological K-theory of a discrete group was introduced in the original formulation of the Baum-Connes conjecture and was later identified as the equivalent K-homology of the universal space of proper actions by the group. As the left hand side of the Baum-Connes conjecture, it displays local feature of an associated elliptic operator, while the right hand side of Baum-Connes accommodates higher indices and displays global feature of the operator. In this lecture we will review various notions of K-homology and introduce a pairing of the topological K-theory with cyclic cocycles coming from the group algebra and the local index formula computing the pairing. This is joint work with Paulo Carrillo-Rouse and Bai-Ling Wang.

Michael Whittaker (University of Glasgow)
A geometric fundamental class for Smale spaces

A few years back, Smale spaces were shown to exhibit noncommutative Poincaré duality (with Jerry Kaminker and Ian Putnam). The fundamental class was represented as an extension by the compacts. In current work we describe a Fredholm module representation of the fundamental class. The proof uses delicate approximations of the Smale space arising from a refining sequence of (open) Markov partition covers. I hope to explain all these notions in an elementary manner. This is joint work with Dimitris Gerontogiannis and Joachim Zacharias.

Wilhelm Winter (University of Münster)
Regularity properties for amenable C^*-algebras and topological dynamics

Finite nuclear dimension, $\mathcal{Z}$-stability, and strict comparison are regularity properties which play a decisive role in the structure and classification theory of simple nuclear C^*-algebras. In this talk I will give an overview of how these properties can be interpreted at the level of topological dynamical systems, and how the two viewpoints can be unified in the presence of C^*-diagonals. For the latter, I will focus in particular on the notion of diagonal dimension, developed in joint work with Kang Li and Hung-Chang Liao.

Makoto Yamashita (University of Oslo)
Homology and K-theory of dynamical systems

A theory of homology for étale groupoids was developed by Crainic and Moerdijk based on simplicial structure of nerves of groupoids, as a companion to Haeflier's theory of cohomology for groupoids. We relate this to another (co)homology of groupoids, namely the operator K-groups of the associated convolution algebra, when the base is totally disconnected. Such a connection was conjectured by Matui through his study of Cantor dynamical systems. Our proof is based on the triangulated categorical structure of groupoid equivariant KK-theory, following the categorical approach to the Baum-Connes conjecture by Meyer and Nest. Similar ideas allow us to prove that Putnam's homology theory for Smale spaces agrees with groupoid homology (without total disconnectedness assumption). Based on joint works with Valerio Proietti.

Schedule Noncommutativity in the North
14 - 18 March 2022

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30 – 9:00	Registration				
9:00 – 9:30	Wilhelm Winter	David Kyed	Hang Wang	Adam Skalski	Michael Whittaker
9:30 – 10:00					
10:00 – 10:30	Coffee	Coffee	Coffee	Coffee	Coffee
10:30 – 11:00	Tatiana Shulman	Jens Kaad	Walter Van Suijlekom	Makoto Yamashita	James Gabe
11:00 – 11:30					
11:30 – 12:00	Kevin Brix	Erik Habbestad	Eske Ewert	Arkadiusz Bochniak	Mikael Rørdam
12:00 – 12:30	Marzieh Forough	Petr Naryshkin	Rui Dong	Diego Martinez	
12:30 – 13:00	Lunch break	Lunch break		Lunch break	
13:00 – 13:30					
13:30 – 14:00					
14:00 – 14:30	Karen Strung	Sanaz Pooya		Magdalena Musat	
14:30 – 15:00		Julian Kranz			
15:00 – 15:30	Coffee	Coffee		Coffee	
15:30 – 16:00	Hannes Thiel	Franz Luef		Matthew Kennedy	
16:00 – 16:30					
16:30 – 17:00	Reception			Problem	
17:00 – 17:30				Session	
17:30 – 18:00					
18:30 – ….		Dinner in restaurant Hyllan			

Invited talks
Contributed talks
Problem session
Breaks and social events

Schedule

Restaurants and Wednesday afternoon activities

There are some restaurants at convenient distance for lunch.

- Wijkanders
- OOTO
- Kårrestaurangen (run by the student association; cheaper and with a salad buffet included)

For Wednesday afternoon we will reserve the rooms MVL21, MVL22 and MVL23 for discussions from 2 to 5 pm. Alternatively, you can explore the city and visit for example the following places.

- Slotskogen, a large park area easily reachable by tram 6 getting of at Linnéplatsen. Here, in the natural history museum you can see one of the few mounted blue whales in the world.
- Botaniska trädgård, the botanical garden of Gothenburg, accessible by tram 7 or 8.
- Göteborgs konstmuseum, the city's art museum, lying at walking distance between university and old town.

Map