

**ABSTRACTS FOR WORKSHOP ON  
ANALYSIS, NONCOMMUTATIVE GEOMETRY,  
AND OPERATOR ALGEBRAS  
CHALMERS UNIVERSITY OF TECHNOLOGY AND  
UNIVERSITY OF GOTHENBURG  
12-16/6-2017**

**Branimir Cacic (University of New Brunswick),**  
*Riemannian principal bundles in unbounded KK-theory*

Unbounded KK-theory has rapidly emerged as a natural framework for accommodating submersions and fibrations in the framework of spectral triples as noncommutative manifolds. Applications of noncommutative geometry to mathematical physics motivate the precise translation of the notion of principal bundle into the framework of spectral triples, but substantial progress has only been made in the case of noncommutative torus bundles. In this talk, I'll discuss how (commutative) Riemannian principal bundles with not necessarily Abelian compact structure groups manifest themselves canonically in terms of spectral triples and unbounded KK-theory under minimal hypotheses; if time permits, I'll also discuss extensions to noncommutative principal bundles arising from theta-deformation and possible implications for noncommutative-geometric gauge theory. This is joint work with Bram Mesland.

**Tyrone Crisp (Radboud University Nijmegen),**  
*Descent of operator modules*

Operator modules are a useful generalisation of both Hilbert space representations and Hilbert  $C^*$ -modules. Operator modules can be induced from a subalgebra  $A$  up to an operator algebra  $B$  using a tensor product construction due to Haagerup. This talk will be concerned with the problem of recognising these induced modules among all  $B$ -modules. I will explain how this problem is related to (in fact, dual to) the question of 'imprimitivity' studied by Mackey, Rieffel, and others; and I will present a solution in the case where  $A$  is a  $C^*$ -algebra with the weak expectation property. I will also discuss a potential application in representation theory.

**Robin Deeley (University of Hawai'i),**  
*The structure of the stable  $C^*$ -algebra of certain Smale spaces*

The stable  $C^*$ -algebra of a Smale space is obtained from the étale groupoid associated to the stable equivalence relation. When the original system is mixing it is simple, separable, nuclear, and stably finite. I will outline the construction of an explicit inductive limit decomposition of the stable  $C^*$ -algebra when the stable sets are totally disconnected. From this, one can often compute the  $K$ -theory of the stable algebra. The talk will be example based. In particular, no knowledge of Smale spaces is required. This talk is based on joint work with Allan Yashinski.

**Heath Emerson (University of Victoria),**  
*Statistics of free groups, topological Markov chains and (twisted) noncommutative geometry*

An examination of the case of a free group acting on its boundary reveals an interesting connection between the Rapid Decay property for the group, and some

of the spectral estimates needed to carry out Connes' proposal for doing Type III noncommutative geometry with twisted spectral triples, for the crossed-product of the group acting on its boundary. We will describe work in progress designed to establish a twisted noncommutative geometry structure on the  $C^*$ -algebras of hyperbolic groups acting on their boundaries with particular focus on the problem of finding the topological significance of KMS states in these and similar such situations. Cuntz-Krieger algebras supply a similar situation, and we discuss them as well.

**Victor Gayral (l'Université de Reims Champagne-Ardenne),**  
*Dixmier traces, zeta functions, heat kernels and an application to pseudodifferential operators*

The aim of this (old) work (joint with Fedor Sukochev) is to explain the connections between Dixmier traces, zeta-functions and traces of heat semigroups beyond the dual of the Macaev ideal and in the general context of semifinite von Neumann algebras. One of our main discovery is the use of an extrapolation functor to study these questions. As an application of our results in the context of Hörmander-Weyl pseudo-differential calculus on  $\mathbb{R}^n$ , we show that the Dixmier trace of a pseudo-differential operator coincides with the 'Dixmier integral' of its symbol.

**Olof Giselsson (Chalmers University of Technology and University of Gothenburg)**  
*Shilov boundary for holomorphic functions on a quantum matrix ball*

The Shilov boundary of a compact Hausdorff space  $X$  relative to a uniform algebra  $\mathcal{A}$  in  $C(X)$  is the smallest closed subset  $K \subset X$  such that every function in  $\mathcal{A}$  achieves its maximum modulus on  $K$ . This notion is encountered, in particular, in the theory of analytic functions in relation to the maximum modulus principle. We will be interested in its non-commutative analog. The latter was introduced by W. Arveson.

In the middle of 1990s, within the framework of the quantum group theory, L.Vaksman and his coauthors started a "quantisation" of bounded symmetric domains. One of the simplest of such domains is the matrix ball  $\mathbb{D} = \{z \in M_m : zz^* < I\}$ , where  $M_m$  is the algebra of complex  $m \times m$  matrices. The Shilov boundary of  $\mathbb{D}$  relative to the algebra of holomorphic functions in  $C(\overline{\mathbb{D}})$  is the set of unitary  $m \times m$ -matrices. In this talk I will discuss the Shilov boundary ideal for the  $q$ -analog of holomorphic functions on the matrix ball. This is a joint work with L.Turowska, O.Bershtein and D.Proskurin.

**Maria Paula Gomez-Aparicio (Université Paris-Sud 11),**  
*The Baum-Connes Conjecture and Oka principle*

The Baum-Connes conjecture with coefficients is still open for higher rank Lie groups. These groups satisfy a strong version of property (T) introduced by Vincent Lafforgue that prevents one to apply the methods that have been used so far to prove the Baum-Connes conjecture. For higher rank Lie groups, one can state a version of Oka principle in noncommutative geometry following ideas of Bost that implies the Baum-Connes conjecture for those groups. In this talk, I will explain this statement using some group Banach algebras defined using tensor products by some finite dimensional representation, say  $\rho$ . I will then show that for complex Lie groups using the Plancherel formula, this algebras decompose into some algebra defined by the action of a weight belonging to the convex hull of the action of the Weyl group on the highest weight of  $\rho$ . This will suggest the construction of continuous families of algebras which interpolate between the "twisted" algebras

defined by  $\rho$ .

**Nigel Higson (Pennsylvania State University),**

*C\*-algebras and the uniform admissibility of the discrete series*

Harish-Chandra's tempered dual of a real reductive group is the closed set in the unitary dual that supports the regular representation; in other words it is the spectrum of the reduced group  $C^*$ -algebra. Many aspects of Harish-Chandra's theory of tempered representations fit very naturally with  $C^*$ -algebra theory and noncommutative geometry. The main results of tempered representation theory are easy to express in  $C^*$ -algebra terms, and noncommutative-geometric arguments explain very succinctly the general form of tempered representation theory, as I shall begin by outlining. I shall then focus on the "uniform admissibility property" of the discrete series. I shall describe what this property is, why it is important, how it can be approached through  $C^*$ -algebras and noncommutative geometry, and what the possible consequences of this approach might be.

**Jens Kaad (University of Southern Denmark),**

*On a theorem of Kucerovsky for half-closed chains*

Kucerovsky's theorem provides conditions for verifying that an unbounded Kasparov module is an unbounded Kasparov product of two unbounded Kasparov modules. When working with open manifolds from a noncommutative geometry point of view, the natural objects to consider are the half closed chains instead of the usual unbounded Kasparov modules. In this talk I will discuss how to extend Kucerovsky's theorem to the setting of half closed chains. On our way we shall see how to construct unbounded modular cycles (twisted unbounded Kasparov module) out of half closed chains and this procedure is in fact a key ingredient for obtaining a suitable extension of Kucerovsky's theorem. The presentation will be spiced up by a few nice geometric examples. The talk is based on joint work with Walter van Suijlekom.

**Matthias Lesch (University of Bonn),**

*Sums of linear operators in Hilbert  $C^*$ -modules*

Given two closed unbounded operators  $A, B$  in a Banach space. There is a rich literature on the problem whether the sum  $A + B$  is closed and regular on the intersection of the domains  $D_A \cap D_B$ . The seminal paper by da Prato and Grisvard (1975) and its successors are mostly motivated by applications to PDE.

Another completely different and quite recent motivation comes from the unbounded picture of  $KK$ -theory. Here, the basic objects are selfadjoint unbounded operators in a Hilbert  $C^*$ -module.

Hilbert  $C^*$ -modules are Banach spaces which retain certain properties of Hilbert spaces but they are lacking many properties which depend on anti-self-duality or the Projection Theorem. Hence Hilbert  $C^*$ -modules are much better than Banach spaces and still far from being as nice as Hilbert spaces; e.g. any  $C^*$ -algebra is naturally a Hilbert  $C^*$ -module.

In my talk I will present a Hilbert  $C^*$ -module version and an elementary proof of a noncommutative Dore-Venni type Theorem for noncommuting operators. The result grew out of discussions with Bram Mesland and is part of an ongoing joint work in progress.

**Yang Liu (Max Planck Institute for Mathematics, Bonn) ,**

*Einstein-Hilbert action on Connes-Landi noncommutative manifolds*

A general question behind the talk is to explore a good notion for intrinsic curvature in the framework of noncommutative geometry started by Alain Connes in the 80's. It has only recently begun (2014) to be comprehended via the intensive study of modular geometry on the noncommutative two tori. In this talk, I will explain how to formulate the Einstein Hilbert action in a functional analytic framework so that it can be extended to the noncommutative setting in a natural way. In the conformal case, the action density (the scalar curvature) was computed in my recent work. The new features consists of certain functions which defines the noncommutative differentials from the noncommutative coordinate. I will explain why such functions deserve more attentions and the geometric meaning of certain functional relations between them.

**Ryszard Nest (University of Copenhagen),**

*Equivariant algebraic index theorem*

We sketch a proof of a  $\Gamma$ -equivariant version of the algebraic index theorem, where  $\Gamma$  is a discrete group of automorphisms of a formal deformation of a symplectic manifold. The particular cases of this result are the algebraic version of the transversal index theorem related to the theorem of A. Connes and H. Moscovici for hypoelliptic operators and the index theorem for the extension of the algebra of pseudodifferential operators by a group of diffeomorphisms of the underlying manifold due to A. Savin, B. Sternin, E. Schrohe and D. Perrot. Joint work with A. Gorokhovsky and N. De Kleijn.

**Shintaro Nishikawa (Pennsylvania State University) ,**

*On the lifting of the Dirac elements in the Higson-Kasparov theorem*

I will talk about the Baum-Connes Conjecture for a-T-menable groups, i.e. the Higson-Kasparov theorem. At the equivariant  $E$ -theory level or in terms of asymptotic morphisms, this theorem can be viewed as an equivariant generalization of the infinite dimensional Bott Periodicity; and the proof goes almost straightforwardly. When we deal with the equivariant  $KK$ -theory, however, some technical issues arise. Although these technicalities were resolved in the paper by Higson and Kasparov, it might be perhaps better to digest what is true at the end of the day. I will describe one of the ways of simply viewing this technical part of the Higson-Kasparov theorem.

**Wojciech Szymanski (University of Southern Denmark),**

*Quantum lens spaces*

I will discuss recent progress on quantum lens spaces and related quantum weighted projected spaces, with special emphasis on the structure of the underlying  $C^*$ -algebras and their relation with graph algebras.

**Robert Yuncken (Université Clermont Auvergne),**

*Twisted spectral triples and pseudodifferential calculus on quantum projective spaces*

Quantum groups and their homogeneous spaces are naturally occurring noncommutative spaces. But incorporating them into Connes-style noncommutative geometry is more difficult than one might hope. For instance, the quantum sphere  $CP_q^1$  is the only quantum homogeneous space for which we have a local index formula (due to Neshveyev-Tuset). In this talk, we will describe some progress on the noncommutative geometry of the higher dimensional quantum projective spaces. En route, we will have to sort out what "regularity" means for a twisted spectral triple. Joint

work with Marco Matassa.