Homework from the

QUTE-EUROPE Summer School 2015 on
Quantum Simulation and Computation:
From fundamentals to applications and implementations

at Hindåsgården, Gothenburg, Sweden, June 21-27, 2015

Here are exercises suggested by our lecturers. Hand in your solutions to your local examiner, at the latest on Monday the 31st of August 2015. The number of credits awarded is up to your local examiner and can of course depend on how many of the exercises you do and your background.

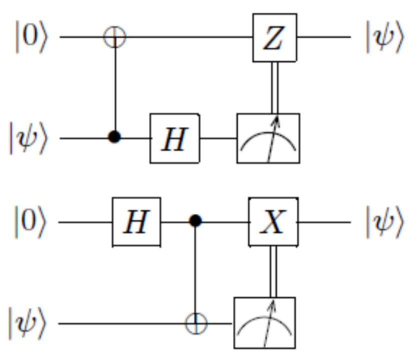
1. Anton Zeilinger's lecture: Derive the relation between the four phases ($\phi_{1,in}$, $\phi_{1,out}$, $\phi_{2,in}$, and $\phi_{2,out}$) in a general (not limited to 50/50) beam-splitter.

2. Seth Lloyd's lecture: see separate page.

3. Steve Girvin's lecture: Verify how to diagonalize the hamiltonian,
   $$H = \omega_c a^\dagger a + \omega_{qb} b^\dagger b + g(ab^\dagger + a^\dagger b)$$
   i.e. how to introduce the dressed qubit/oscillator operators.

4. Barbara Terhal's lecture: see separate page.

5. Elham Kashefi's lecture: Verify and explain in words the action of the following two gate teleportation circuits.



6. Andrew White's lecture: Derive the Hong-Ou-Mandel visibility.
   You already did this during the lecture.

7. Peter Love's lecture: Expand exp((A+B)*t*) and exp(A*t*) exp(B*t*) to second order in *t* and see how they differ. You already did this during the lecture.
   Left to do: Show that the Jordan-Wigner transformation gives you the anti-commutation relations.

8. Matthias Troyer's lecture: Write a quantum circuit to implement the time-evolution exp(-iH*t*) under the Hubbard-U-term Hamiltonian
   $H = U\hat{n}_\uparrow \hat{n}_\downarrow$
   between two spin orbitals.

9. Ken Brown's lecture:
   a) Compare the Cirac-Zoller and Molmer-Sorensen gates for entangling ions in the same trap. In what ways is the Molmer-Sorensen gate more robust?

   b) In the lecture, we examined how qubits can be entangled remotely by entangling the ion state with the emitted photon frequency. Show how this can also be done by entangling the ions with the light polarization.

Homework for Seth Lloyd's lecture

A quantum random access memory accesses information – classical or quantum – in a way that preserves quantum coherence. Consider a qRAM that contains $N = 2^n$ memory slots, addressed by a memory register with n qubits. Suppose that the information in the memory consists of N numbers x_j, each of which can be equal to ± 1. A qRAM call takes as input the memory register state state $|j\rangle$ together with an ancilla $|0\rangle$ and outputs the state $|j\rangle|x_j\rangle$. The qRAM call is coherent in the sense that

$$\sqrt{1/N}\sum_j |j\rangle|0\rangle \longrightarrow \sqrt{1/N}\sum_j |j\rangle|x_j\rangle.$$

The qRAM can also be used to perform the inverse operation $|j\rangle|x_j\rangle \to |j\rangle|0\rangle$

Problem 1: Show how to use the qRAM with the transformation $|x_j\rangle \to x_j|x_j\rangle$ to construct the state

$$\sqrt{1/N}\sum_j x_j|j\rangle.$$

This state encodes N classical bits – the binary data x_j – into a quantum state over $n = \log_2 N$ qubits.

Problem 2: Suppose that $x_{j+p} = x_j$, so that the classical data is *periodic* with period p. What do you think happens when you apply a quantum Fourier transform to the quantum state $\sqrt{1/N}\sum_j x_j|j\rangle$? Compare the application of the QFT to the quantum representation to the application of a classical discrete fast Fourier transform to the classical data x_j. (Hint: the results are very similar, but the QFT requires $\approx n^2$ quantum logical operations while the classical FFT requires $\approx n2^n$ classical logic operations.)

For problem 2, don't worry if you don't know the exact formulae for the effect of the QFT and FFT. Just give a verbal description of what you think happens. If you do know the mathematical description of the QFT and FFT, however, feel free to do the actual calculation.

Problem 3: Suppose that the unknown periodicity p is on the order of $\sqrt{N}$. Give an argument for why you can't find this periodicity classically without sampling on the order of at least $\sqrt{N}$ data.

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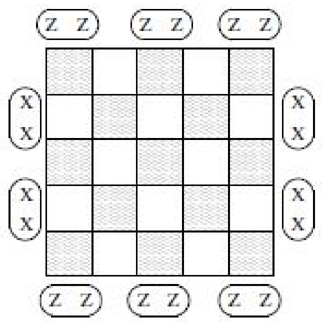
Exercises Lectures on Surface Code Architecture by B.M. Terhal

June 16, 2015

Problem 1: Counting the number of encoded qubits

A stabilizer code on n physical qubits with $n-k$ linearly-independent parity checks encodes k logical qubits. Show why the toric code encodes 2 logical qubits and the surface code encodes only 1 logical qubit by considering the linear dependencies between parity checks. Why does the lattice below (picture taken from Hastings/Geller, arXiv.org: 1408.3379) encode a single logical qubit? For this lattice the physical qubits are on the vertices and a dark plaquette is a weight-4 X-check while a white plaquette is a weight-4 Z-check. The oval weight-2 checks act on the vertices on the boundary.



Problem 2: CNOT by Measurement

Verify that a CNOT gate can be implemented as in the Figure below. What are the Pauli corrections to get the correct CNOT dynamics?

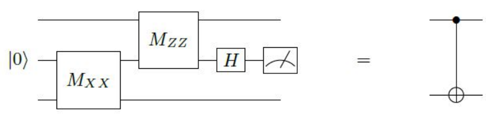


FIG. 10 CNOT via 2-qubit quantum measurements. Here M_{XX} measures the operator $X \otimes X$ etc. The ancilla qubit in the middle is discarded after the measurement disentangles it from the other two input qubits. Each measurement has equal probability for outcome ± 1 and Pauli corrections (not shown, see Eq. (9)) depending on these measurement outcomes are done on the output target qubit.