CURVATURE INEQUALITIES FOR OPERATORS IN THE COWEN-DOUGLAS CLASS OF A PLANAR DOMAIN

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Abstract: Fix a bounded domain Ω in the complex plane $\mathbb{C}$. If an operator T, in the Cowen-Douglas class $B_1(\Omega)$, admits the compact set $\bar{\Omega}$ as a spectral set, then the curvature inequality $\mathcal{K}_T(w) \leq -4\pi^2 S_\Omega(w,w)^2$, where S_Ω is the Sz̈ego kernel of the domain Ω, is evident.

In particular, for any contraction T in $B_1(\mathbb{D})$, we have $\mathcal{K}_T(w) \leq -(1-|w|^2)^{-2} = \mathcal{K}_{U_+^*}(w)$, where U_+^* is the standard unilateral backward shift operator. However, it is easy to construct examples of contractive operators T in $B_1(\mathbb{D})$ for which $\mathcal{K}_T(w_0) = -(1-|w_0|^2)^{-2}$ for some $w_0 \in \mathbb{D}$ but T is not unitarily equivalent to U_+^*. After imposing some "mild" conditions on the class of co-subnormal contractions T in $B_1(\mathbb{D})$, it is shown that if $\mathcal{K}_T(w_0) = -(1-|w_0|^2)^{-2}$ for an arbitrary but fixed point $w_0 \in \mathbb{D}$, then T is unitarily equivalent to U_+^*.

Except when Ω is simply connected, the existence of an operator for which $\mathcal{K}_T(w) = -4\pi^2 S_\Omega(w,w)^2$ for all w in Ω is not known. However, one knows that if w is a fixed but arbitrary point in Ω, then there exists a bundle shift of rank 1, say S, depending on this w, such that $\mathcal{K}_{S^*}(w) = 4\pi^2 S_\Omega(w,w)^2$. We prove that these *extremal* operators are uniquely determined:

If T_1 and T_2 are two operators in $B_1(\Omega)$ each of which is the adjoint of a rank 1 bundle shift and $\mathcal{K}_{T_1}(w) = -4\pi^2 S(w,w)^2 = \mathcal{K}_{T_2}(w)$ for a fixed w in Ω, then T_1 and T_2 are unitarily equivalent. A surprising consequence is that the adjoint of only some of the bundle shifts of rank 1 occur as extremal operators in domains of connectivity > 1. These are described explicitly.