KASS 2013

December 4, 2013, 10.00

Hossein Raufi (Göteborg): *Log concavity for matrix-valued functions, and a matrix-valued Prékopa theorem.*

Abstract: We define what it means for a matrix-valued function to be log concave. After discussing a few simple examples and properties, we introduce a matrix-valued Prékopa theorem. We end by sketching the ideas behind the proof of this theorem, using a weighted, vector-valued Paley-Wiener theorem and positivity properties of direct image bundles.

November 27, 2013, 10.00

Chinh Lu Hoang (Göteborg): *Some generalized capacities and applications. (joint work with Eleonora Di Nezza)*

Abstract: We study generalized capacities on a compact Kähler manifold, where the candidates are normalized between two quasi-plurisubharmonic functions. As an application of these capacities we consider complex Monge-Ampère equations on compact Kähler manifolds with the right-hand-side being smooth outside a complex hypersurface. We prove that there exists a unique solution (up to an additive constant) which is smooth outside the hypersurface, under a condition on the hypersurface.

November 14, 2013, 13.15

Genkai Zhang (Göteborg): *Kaehler metrics on moduli space of projective structures on a surface, joint with Inkang Kim.*

Abstract: We compute the group cohomology of a representation of the fundamental group of a surface in SL(3, R) defining a convex projective structure, and we prove the existence of a Kaehler metric on the moduli space of the projective structures.

October 17, 2013, 13.15

Joaquim Ortega-Cerdà (Universitat de Barcelona): *Fekete points on complex manifolds.*

Abstract: I will present a joint work with Nir Lev from the University of Bar-Ilan. In this work we study the equidistribution of Fekete points in a compact complex manifold. These are extremal points defined through sections of powers of an holomorphic line bundle and they correspond in their most classical setting to the spatial distribution of electrons under the influence of an external field. The equidistribution of the Fekete points towards an equilibrium measure is a result of Berman, Boucksom and Witt. The novelty of our approach is the relationship established between the Fekete points and sampling and interpolation points in holomorphic line bundles. This allows us to estimate quantitatively the equidistribution of the Fekete points.

September 25, 2013, 10.00

Mats Andersson (Göteborg): *Generalized cycles in projective space and local intersection numbers.*

Abstract: Let Z and W be two varieties in projective space. Some years ago, Tworzewski, and independently Gaffney and Gassler, introduced for each point on the set-theoretical intersection of Z and W, a list of non-negative integers, called the local intersection numbers.

We introduce a class of generalized cycles, that contains all analytic cycles. Each generalized cycle has well-defined multiplicity at each point and a well-defined degree. Given two (generalized) cycles Z and W we define a product $Z \bullet W$, which is a generalized cycle with the property that its multiplicities at each point is precisely the local intersection numbers. Moreover, the product respects Bezout's identity. We also discuss the relation to the classical non-proper intersection product. In particular, from $Z \bullet W$ one can get the associated cohomology class on the set-theoretical intersection of Z and W. This is a joint work with D Eriksson, H Samuelsson, E Wulcan and A Yger.

September 23, 2013, 15.15

Mihai Paun (KIAS, Seoul): *Metrics with prescribed Ricci curvature in a conic setting.*

September 9, 2013, 16.15

Samuele Mongodi (SNS, Pisa): *Metric currents and holomorphic chains in Hilbert spaces.*

Abstract: The main part of this talk will be devoted to the definition of metric currents and their fundamental properties: given a metric space X, we can define *currents* on it as multilinear functionals on $n+1$-tuples of Lipschitz functions, satisfying some additional conditions of locality, continuity and boundedness. A complex space admits several metrics, variously related to the complex structure. The complex structure is taken into consideration by a variant of the locality axiom, producing (p,q)-currents, which vanish whenever the number of holomorphic or antiholomorphic differentials is different from (p,q). This setting permSamuele Mongodits to talk about currents in infinite-dimensional spaces, like complex Hilbert spaces, where classical problems concerning positive currents and boundaries of analytic chains can be reformulated and solved. Namely, King's result about the characterization of positive currents holds in Hilbert spaces as well and the boundary problem, first attacked by Harvey and Lawson in the compact case, can be given a positive answer (i.e. every compact finite dimensional maximally complex manifold is the boundary of an analytic chain).

September 9, 2013, 15.15

Sara Lapan (Northwestern University, Evanston): *On the existence of attracting domains for maps tangent to the identity.*

Abstract: One of the guiding questions behind the study of local discrete holomorphic dynamics is: given a germ of a holomorphic self-map of $\mathbb{C}^m$ that fixes a point (say the origin), can it be expressed in a simpler form? If so, then the dynamical behavior of the map can be more easily understood. In general, we want to know how points near the origin behave upon iteration of the map f. More specifically, we want to know when there exists a domain whose points are attracted to the origin under iteration by f and, if such a domain exists, when its points converge tangentially to a given direction. In dimension one, the Leau-Fatou Flower Theorem tells us, among other things, of the existence of such domains. In higher dimensions, Hakim showed that given some assumptions on f and the direction v, a domain of attraction whose points converge to the origin tangentially to v does exist. In this talk, we will consider collections of maps that do not satisfy some of the assumptions in Hakim's theorem. In particular, we will discuss different types of germs of holomorphic self-maps of $\mathbb{C}^2$ and $\mathbb{C}^3$ that fix the

origin, focusing on when there exists (or does not exist) an invariant attracting domains whose points converge to the origin tangentially to a given direction.

August 28, 2013, 10.00

David Witt Nyström (Göteborg):*Restricted volumes and the augmented base locus.*

Abstract: Let L be a holomorphic line bundle on a projective manifold X. The augmented base locus of L is an analytic subset of X which in a certain sense captures the "bad" behavior of L: away from the augmented base locus the line bundle acts like its ample. Thus the augmented base locus is also referred to as the non-ample locus. Recall that the restricted volume of L along a subvariety V measures the asymptotic growth of the dimension of the vector spaces $H^0(X|V, kL)$, which consists of the sections of kL (i.e. the k:th tensor power of L) on V that extend to X. Ein-Lazarsfeld-Mustata-Nakamaye-Popa proved that the augmented base locus is equal to the union of all irreducible subvarieties along which the restricted volume of L is zero. We will discuss a new proof (supposed that L is big) that only uses pluripotential methods. Thus it also proves the analogue statement in the more general setting of big (1,1)-classes.

June 20, 2013, 10.00

Jakob Hultgren (Göteborg): *Invitation to toric geometry.*

Abstract: A toric variety is a variety containing a complex torus as a dense subvariety such that the action of the torus on itself extend to the whole variety. Examples include $\mathbb{P}^n$ and other equivariant compactifications of $\mathbb{C}^n$. In a lot of ways toric varieties are extremely simple and special cases of algebraic varieties, but nevertheless a lot of phenomenon from algebraic geometry carry over to toric geometry. My impression is that the most common application of toric geometry is as a way of illustrating phenomenon from algebraic geometry in a somewhat concrete setting, and ultimately, as a possible testing ground for conjectures in algebraic geometry.

The idea is that we will go through some basic theory of toric varieties, i.e. the fans and the polytopes, and, depending in time, look at examples of some phenomenon from algebraic geometry. I have prepared material on Cartier, Q-Cartier & non-Cartier divisors as well as basepoint free/non-basepoint free and ample/non-ample divisors. We will see how far we get.

May 22, 2013, 10.00

Bo Berntsson (Göteborg): *The openness conjecture for projective manifolds.*

Abstract: The opennes conjecture was formulated by Demailly and Kollàr in 1999 (`http://arxiv.org/abs/math/9910118`). It says roughly that the set of plurisubharmonic functions, u, such that $e^{(-u)}$ is locally integrable near a given point is open. I will prove a global version of this for positively curved metrics on projective manifolds. As a consequence the original openness conjecture holds for functions with isolated singularities, see (`http://arxiv.org/abs/1305.0544`). (The full openness conjecture was proved in dimension 2 by Favre and Jonsson, (`http://arxiv.org/abs/math/0406109`)).

May 15, 2013, 10.00

Per Salberger (Göteborg): *Heath-Browns determinantmetod och Mumfords geometriska invariantteori.*

Abstract: Heath-Brown's p-adic determinant method is used to count rational points on hypersurfaces and was extended to subvarieties of codimension >1 in projective space by Broberg and the author. The determinants that occur give rise to embeddings of Hilbert schemes in Grassmannians, which enables the use of techniques from geometric invariant theory. We describe some Diophantine applications and an unexpected link to the theory of Donaldson and Tian on Kähler metrics of constant scalar curvature.

March 13, 2013, 10.00

Elizabeth Wulcan (Göteborg) *On a representation of the fundamental class of an ideal due to Lejeune-Jalabert.*

Abstract: Given a holomorphic function f the classical Poincare-Lelong formula asserts that the current of integration along the variety of f, counted with multiplicites, equals the residue current of f times the form df. I will discuss (a current version of) a generalization of the P-L formula due to Lejeune-Jalabert: the fundamental cycle of a Cohen-Macaulay ideal of holomorpic germs admits a representation as a residue constructed from a free resolution of the ideal times a certain differential form coming from the resolution.

I will give an explicit description of this differential form in the case of monomial ideals. It turns out that in a certain sense the fundamental cycle is captured already in this form.

March 6, 2013, 10.00

David Witt Nyström (Göteborg): *On the inverse logarithmic potential problem in the plane.*

Abstract: Let U be a compact set in the plane. If we integrate z^k over U we get a complex number, called the k:th complex moment of U. By putting these together we get a map from the compact sets in the plane to the set of sequences of complex numbers. The inverse logarithmic potential problem concerns the "inverse" of this map. It is in fact easy to see that the map is neither surjective nor injective, but if one restricts the domain of definition to some smaller class of sets one might hope to achieve injectivity. Results in this direction are due to e.g. Novikov, Shahgholian and Gardiner-Sjödin. There are also various local existence results which tells us that there are certain directions in which we can perturb the complex moments and get a correspond perturbation of the set. Put differently one identifies line segments in the image of the moment map. I will discuss a recent such result due to Julius Ross and myself. I hope to explain the main ideas of the proof, which involves perturbation theory for holomorphic curves attached to a totally real submanifold.

February 27, 2013, 10.00

Hossein Raufi (Göteborg): *Singular hermitian metrics on holomorphic vector bundles, continued.*

Abstract: Last week we introduced the notion of singular hermitian metrics on holomorphic vector bundles. In this seminar we continue developing their properties, with special emphasis on regularisation. After a quick repetition of the main concepts and results of the previous seminar, we define what it means for a singular hermitian metric to be strictly negatively curved in the sense of Nakano, and show that it is possible to regularise such metrics with a sequence of smooth, strictly Nakano negative metrics.

February 20, 2013, 10.00

Henri Guenancia (Paris VI, ENS): *Semi-stability of the tangent bundle of singular varieties with ample canonical bundle.*

Abstract: If X is a smooth projective variety with K_X ample, then it follows from Aubin's theorem that T_X admits a Hermite-Einstein metric, and therefore a theorem of Kobayashi shows that T_X is semi-stable with respect to K_X. We will explain the approach of Enoki who generalized this result to the case where X has canonical singularities,

and underline the difficulties and possible strategies when X has log-canonical singularities.

February 13, 2013, 10.00

Hossein Raufi (Göteborg): *Singular hermitian metrics on holomorphic vector bundles.*

Abstract: We present a notion of singular hermitian metric on holomorphic vector bundles, and compare it with the corresponding concept on line bundles. We then introduce what it means for a singular hermitian metric to be curved in the sense of Griffiths and Nakano, and proceed to define the curvature tensor locally. We end by discussing the regularisation of such metrics.

January 16, 2013, 10.00

Martin Sombra (ICREA & Universitat de Barcelona): *An arithmetic Nullstellensatz over an affine variety.*

Abstract: I will report on joint work with Carlos D'Andrea (Barcelona) and Teresa Krick (Buenos Aires) on bounds for the degree and the height of the polynomials arising in some problems in effective algebraic geometry including the implicitization of rational maps and the effective Nullstellensatz over a variety.

Our treatment is based on arithmetic intersection theory in products of projective spaces and extends to the arithmetic setting constructions and results due to Jelonek. A key role is played by the notion of canonical mixed heights of multiprojective varieties. We study this notion from the point of view of resultant theory and establish some of its basic properties.
