Inverse problems with generative neural priors

Motivation and background

Inverse problems refer to the task of reconstructing parameters characterizing a system under investigation from indirect observations of the system. This type of problems arise in several areas of science and engineering, for example in geo-science, material science, astrophysics, and medical imaging. Examples in the latter category are X-ray Computed Tomography (CT), Magnetic Resonance Imaging (MRI), and Positron Emission Tomography (PET). Lately, data-driven methods (i.e. methods based on machine learning) have become increasingly powerfull and popular for solving inverse problems; for a review, see, e.g., Arridge et al., 2019. Nevertheless, when solving an inverse problem it can be advantageous to take a probabilistic perspective, meaning that one does not only reconstruct a point estimate of the sought parameters but instead reconstruct the posterior distribution conditioned on the observed data. This would make it possible to sample from the posterior and hence one could pose and answer probabilistic questions about the reconstruction, for example questions related to error bounds and uncertainty quantification.

In the recent work Adler and Öktem, 2018, the authors present a methodology for training generative deep neural network which, given observation data, samples from the posterior distribution. While the results are very promising, the methodology has a few drawbacks: i) the methodology needs paired training data, i.e., ground truth estimates of the parameters sought (the image, or the signal, or the coefficients, etc.) paired with corresponding data. This is an issue since in certain cases, and in particular so in medical imaging, the data is often discarded after a reconstruction is obtained, which limits the available training data. Moreover, ii) since the methodology is completely data-driven it does not use any domain-specific knowledge, i.e., it does not use the (massive amount of) mathematical modelling and research that has often been put into the specific field. In fact, in many areas there are relevant and accurate mathematical descriptions for how the data is generated, and the difficulty in sampling from the posterior comes from the difficulty to specify a relevant prior. As an example, consider MRI imaging of the head: there are good mathematical models for how the data is generated, but we do not have any good mathematical descriptions on how cross sections of the human brain looks.

Outline of project

In this work, the idea is to combine machine learning with Markov Chain Monte-Carlo (MCMC) methods to obtain a methodology that can sample from the posterior distribution and that, at least partially, overcomes these issues. More precisely, the idea is to train a generative neural network to generate samples from a prior distribution of objects of interest. This can be done without an explicit notion of an inverse problem, as long as the training of the generative network is done on objects that are representative for the type of objects under investigation in the inverse problem. If so, then this trained neural network will be a transformation from a simple distribution, such as white noise, into a meaningful prior which can be used in the inverse problem. This network based prior will then be combined with a motivated mathematical model for the observation procedure, incorporating domain knowledge, and put into an efficient MCMC-framework, which together generates a way to sample from the posterior. In this way, one can combined the power of data-driven methods with powerful mathematical tools and physical models for the inverse problems. Moreover, since the training of the generative model is done only in order to learn the prior distribution, the training does not need paired training data.

Applicant profile

Two students with strong backgrounds in mathematics and in programming.

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References

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[2] Simon Arridge et al. "Solving inverse problems using data-driven models". In: *Acta Numerica* 28 (2019), pp. 1–174.