Convergence study of the Domain Decomposition Finite Element/Finite Difference Method for time-dependent Maxwell's equations

Master's Thesis Project

Background

In the project we will study convergence of the Domain Decomposition Finite Element/Finite Difference Method for time-dependent Maxwell's equations developed in [2, 3, 4], for the real-life model problems in three dimensions.

The problem of numerical solution of time-dependent Maxwell's equations for electric field arises in many electromagnetic applications including biomedical imaging and specifically, microwave biomedical imaging. Microwave imaging is a complement to the X-ray technique with the goal to produce images of internal structure of body tissues.

This Master's project is motivated by the major challenges in microwave imaging for the medical diagnostics, such as

- high contrast reconstruction of malign tumors
- detection of small blood clots in early stages of stroke.

The above mentioned problems can be modeled by the system of Maxwell's equations for electric field in time domain which we will study in the project. The project will have close connection to the research conducted by the group of biomedical imaging at the Department of Electrical Engineering at Chalmers, see Figure 1 for different research directions where the current Master project can have applications.

Description of the project

In this project we will study convergence of the domain decomposition method for time-dependent Maxwell equations for electric field $E(x,t)$, and our model problem will be

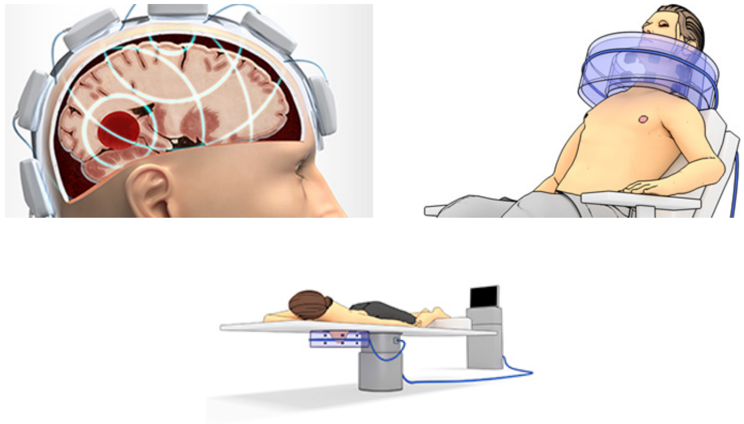

Fig. 1 Biomedical Imaging at the Department of Electrical Engineering at CTH, Chalmers. Left: setup of Stroke Finder and right: microwave hyperthermia in cancer treatment. Below: breast cancer detection using microwave tomography.

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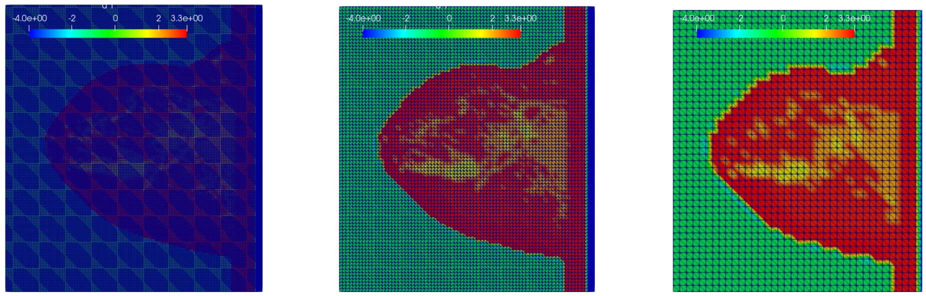
\begin{aligned}
\frac{1}{c^2}\varepsilon_r(x)\frac{\partial^2 E}{\partial t^2} + \nabla \times \nabla \times E = -\mu_0 \sigma(x)\frac{\partial E}{\partial t}, \\
\nabla \cdot (\varepsilon E) = 0, \\
E(x,0) = f_0(x), \quad \frac{\partial E}{\partial t}(x,0) = 0, \;\; x \in \mathbb{R}^3, t \in (0,T]
\end{aligned}
\tag{1}
$$

in the non-magnetic medium in three dimensions. Here, the electric field $E(x,t) = (E_1,E_2,E_3)(x,t), x \in \mathbb{R}^3$ changing in the time interval $t \in [0,T]$ under the assumption that the dimensionless relative magnetic permeability of the medium is $\mu_r \equiv 1$. In (1) ω is the angular frequency, the functions $\varepsilon_r(x) = \varepsilon(x)/\varepsilon_0, \; \mu = \mu_r \mu_0 := \mu_0$ (since $\mu_r = 1$) and $\sigma(x)$ are relative dielectric permittivity, permeability and electric conductivity functions, respectively, and $c = 1/\sqrt{\varepsilon_0 \mu_0}$ is the speed of light in free space.

Existing C++/PETSc [6] code for solution of equation (1) using the domain decomposition FE/FD methods in 2D and 3D is available for testing [8]. Code is implemented with homogeneous and absorbing boundary conditions.

Purpose of the project

The main purpose of this project is perform study of numerical convergence of the Domain Decomposition Finite Element/Finite Difference Method (DD FE/FD method) for time-dependent Maxwell's equations which is implemented in C++/PETSc [6]. The specific goals of this project are:



h= 0.1 h= 0.2 h=0.4

Fig. 2 *Distribution of the relative dielectric permittivity ε_t in the 2D slices of breast phantoms of [7] on different finite element meshes with mesh sizes h (cm).*

- Study existing numerical techniques for solution of Maxwell's equations.
- Study and modify the existing C++/PETSc code for solution of (1) in three dimensions via DD FE/FD method for values of function $\varepsilon_r(x)$ and $\sigma(x)$ related to the real-life applications (see for example [1] for values of $\varepsilon_r(x)$ and $\sigma(x)$ used in microwave hyperthermia and [7] for these values in breast phantoms, see also Figure 2).
- Study numerical convergence of the DD FE/FD method.

The Master's thesis can lead further for PhD position for the project ' 'Efficient algorithms for microwave imaging based on a new non-local optimization approach", see description of the project in [5].

Contact information

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References

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