Optimal test planning for HCF testing
using Bayesian statistics and entropy minimization

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High Cycle Fatigue testing

- A mechanical component is subjected to cyclical stress of some amplitude to see if it breaks or not.
- Testing is destructive for the components, and takes a long time.
- The goal is to find the fatigue limit distribution using as few tests as possible.

High Cycle Fatigue testing

- **Traditional test protocol**:
  If the last tested component broke, decrease the amplitude a fixed step, otherwise increase it.

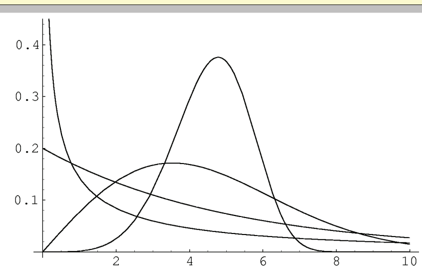
High Cycle Fatigue testing

- **Traditional test protocol**:
  If the last tested component broke, decrease the amplitude a fixed step, otherwise increase it.
- **Proposed test protocol**:
  Use the amplitude that is expected to give as much information as possible about the fatigue limit distribution.

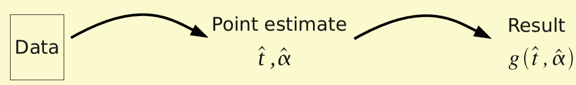
Weibull distribution

$$f(x|t,\alpha)=\frac{\alpha}{t}\left(\frac{x}{t}\right)^{\alpha-1}e^{-(x/t)^{\alpha}}$$

Scale t Shape α



Classical estimation



Classical estimation

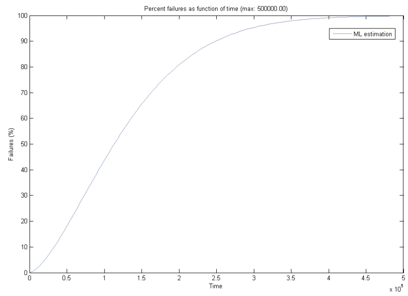
Data → Point estimate $\hat{t}, \hat{\alpha}$ → Result $g(\hat{t}, \hat{\alpha})$

Bayesian estimation

Data → Distribution $P(t, \alpha)$ → Result distribution $P(g(t, \alpha))$

Weibull distribution

Classical solution: Estimate *t* and α from data



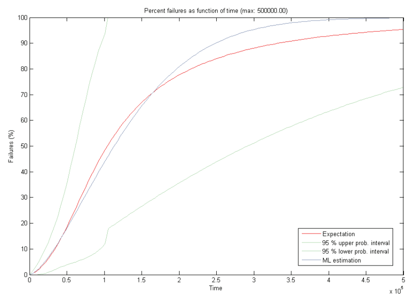
t=144000 a=1.52

Bayesian Statistics

- More robust in case of few data
- Takes into account chance variation in the data
- Gives a measure of the uncertainty
- Not just a point estimate, but a probability distribution over the parameters
- All questions need to be integrated over that distribution

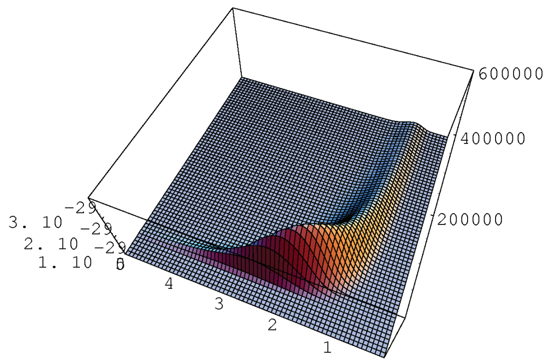
Weibull distribution

Bayesian solution: Integrate over all values of *t* and α



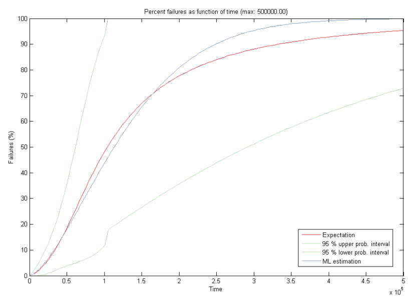
Bayesian Statistics

Posterior distribution of *t* and α



Weibull distribution

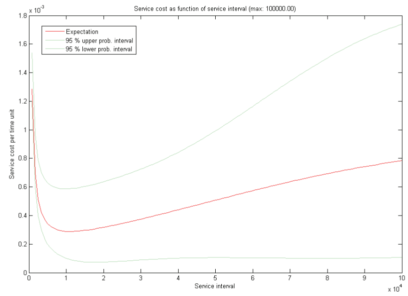
The risk of failure at different service intervals



$$g(t,a)=1-e^{-(x/t)^{a}}$$

Weibull distribution

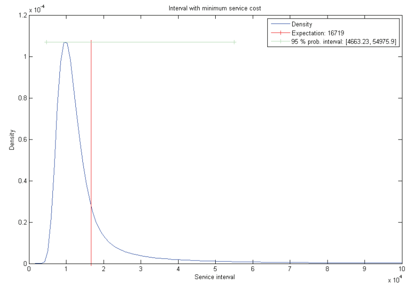
The expected cost of different service intervals



$$g(t,\alpha)=\frac{e^{-(x/t)^{\alpha}}c_{service}+(1.0-e^{-(x/t)^{\alpha}})c_{failure}}{t\Gamma(1+1/\alpha)\Gamma(1+1/\alpha,(x/t)^{\alpha})+e^{-(x/t)^{\alpha}}x}$$

Weibull distribution

The service interval with minimum cost



$$g(t,a)=\langle Numerical \rangle$$

Optimal test planning

- The fatigue limit can be modeled by the Weibull distribution
- SICS has developed a tool for Bayesian integration of the Weibull distribution

$$\int_{t,\alpha} g(t,\alpha)P(t,\alpha|X)$$

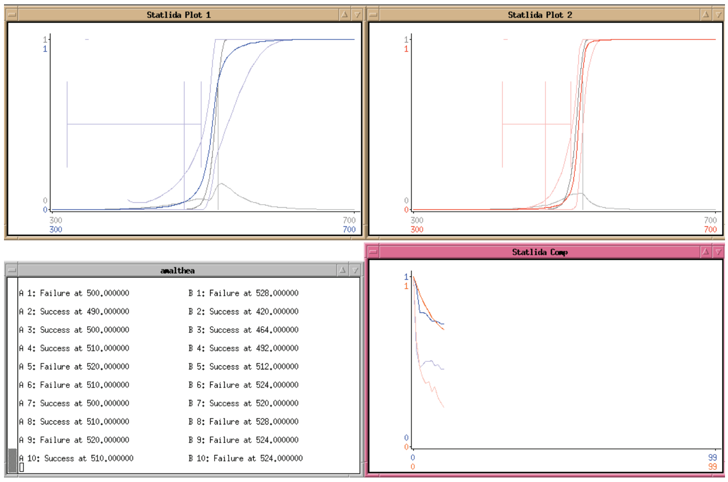
- The entropy of the posterior distribution can be expressed as such an integral

$$\int_{t,\alpha} \log P(t,\alpha|X)P(t,\alpha|X)$$

Optimal test planning

- For every amplitude, calculate the expected entropy decrease of the posterior distribution, and select the maximum

$$max_A(H(t,\alpha|X)-P(+A|X)H(t,\alpha|X,+A)-P(-A|X)H(t,\alpha|X,-A))$$



Traditional protocol to the left, proposed protocol to the right.
Already after 10 tests the proposed protocol is significantly more certain.

Conclusion

- By using **Bayesian life time analysis** together with an **entropy based test design** it is possible to **significantly decrease** the number of required tests.