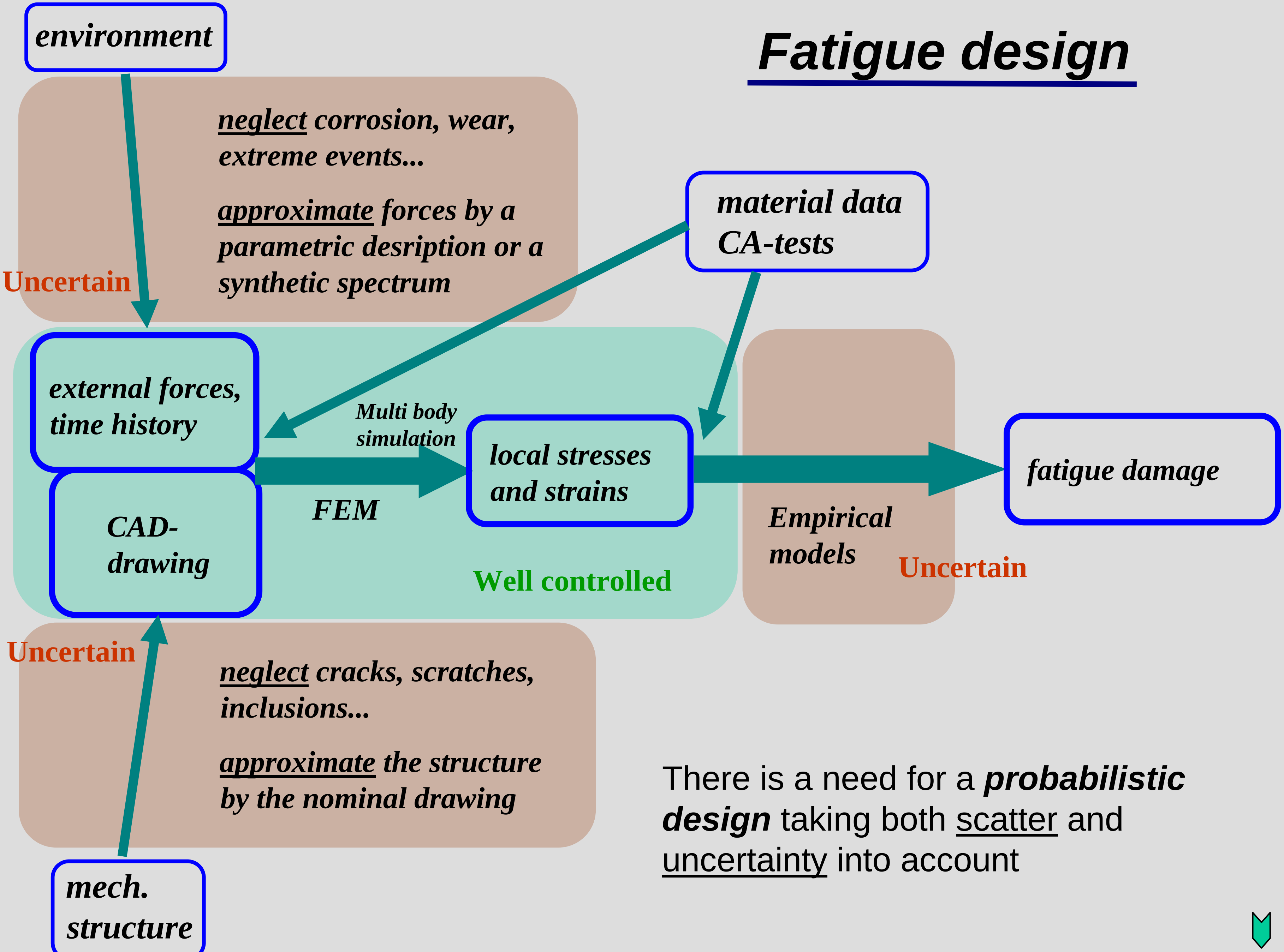
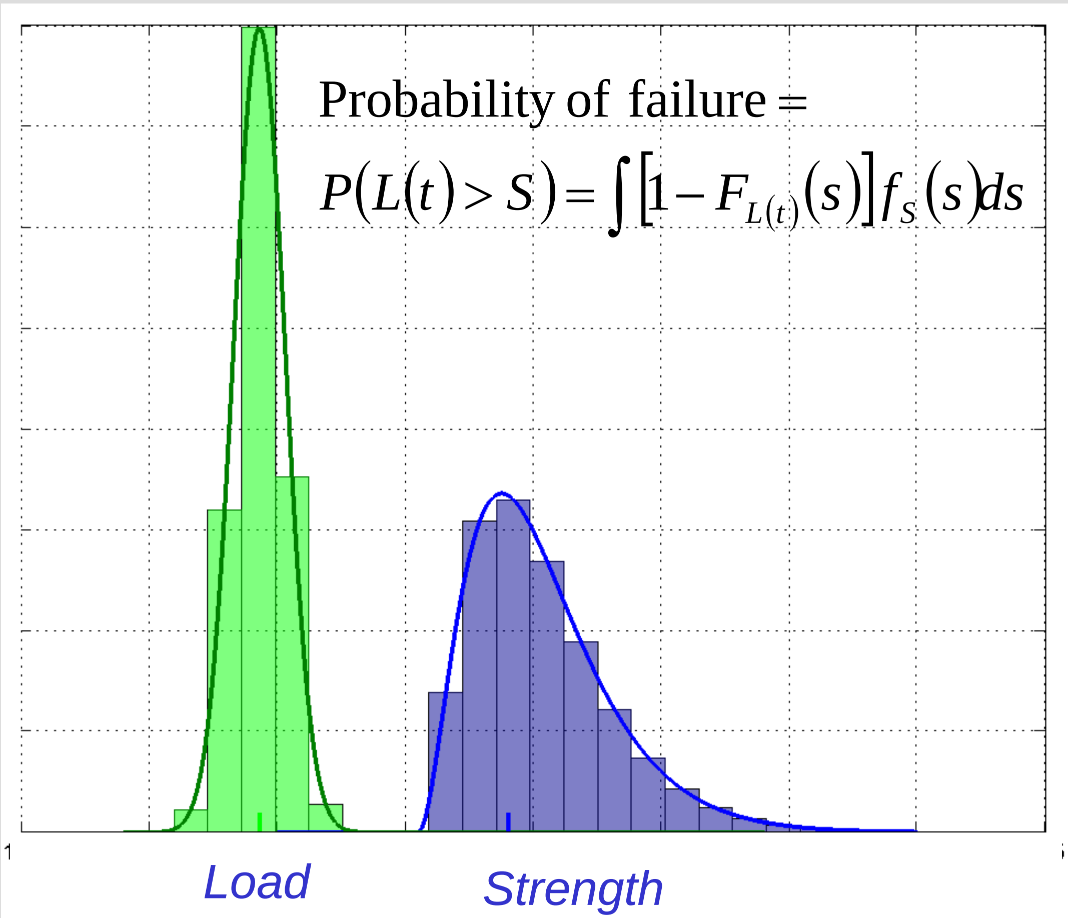
Fatigue design



There is a need for a ***probabilistic design*** taking both scatter and uncertainty into account

The load/strength model



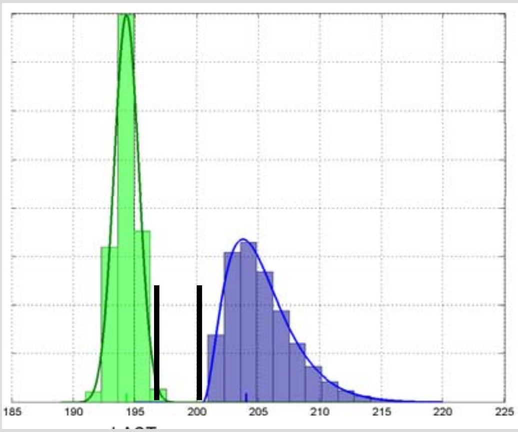
Variability among users

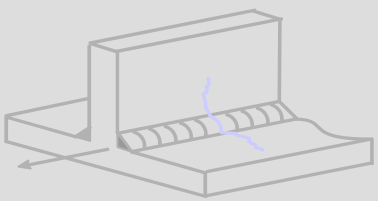
Variability in product quality

Three levels of complexity:

1. *probabilistic design based on complete input distributions.*
2. *methods based only on second moments*
3. *Worst case approaches*

Worst case approaches





Trucks:	*The most severe customer is compared to the weakest vehicle configuration.*
Air engines:	*The $\mu-3\sigma$ material strength is used in calculations and the final result is used with a proper safety factor.*
Welded prod.:	*Standardized load/strength curves are used that correspond to low percentiles in the strength distribution.*
Buildings:	*Partial safety factors are used to get the worst cases for load variables and weakest cases for strength variables. These are combined to an overall reliability measure.*

Weaknesses of these approaches:

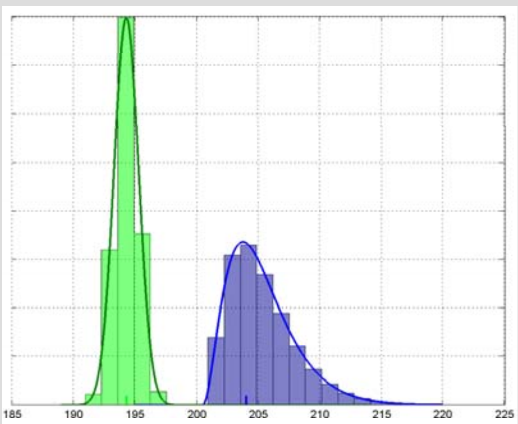
They are highly based on subjective <u>engineering judgement</u>

many variational sources and model errors can not be controlled

Probabilistic design based on input distributions

$$\text{Probability of failure} =$$

$$P\big(L(t) > S\big) = \int \big[1 - F_{L(t)}(s)\big] f_S(s)\, ds$$



- *Identify all <u>scatter and uncertainty</u> components*
- *Establish mathematical models for the variation*
- *Calculate the probability of failure using …*

Numerical integration

or

Monte Carlo simulations

In engineering practise the use of the <u>Monte Carlo method</u> is growing fast.

But, it is usually based on guesses, since the knowledge of the input distributions is highly limited.

A second moment approach

Each variational source is usually only known by

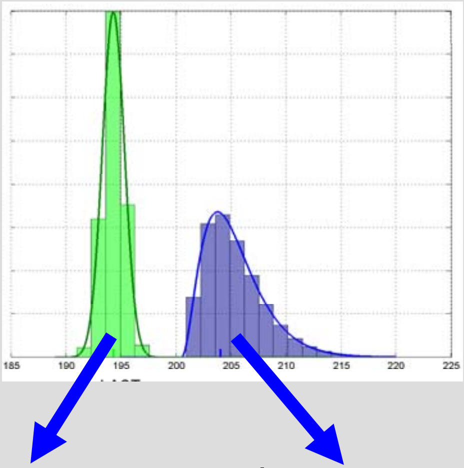
1) *its nominal value as a specified value or an estimated mean*
2) *its dispersion as a tolerance interval, a judgement about its extreme limits or its estimated variance.*

We try to establish simple tools for a <u>first order, second moment probability method</u> based on existing knowlegde.

The basic tool is the <u>Gauss approximation formula</u>

$$\sigma^2_{f(x)} = \sum_i c_i^2 \sigma^2_{x_i} + \sum \mathrm{Cov}$$

In case of fatigue life we apply this to the log-transformed load and strength variables.



$$\left\{\mu_{\ln L}, \sigma^2_{\ln L}\right\} \qquad \left\{\mu_{\ln S}, \sigma^2_{\ln S}\right\}$$

A reliability index

$$\sigma_{\ln L}^2 = \sum_i c_i^2 \sigma_{x_i}^2 + \sum \mathrm{Cov}$$

$$\sigma_{\ln S}^2 = \sum_j c_j^2 \sigma_{x_j}^2 + \sum \mathrm{Cov}$$

$$I = \frac{\mu_{\ln(S)} - \mu_{\ln(L)}}{\sqrt{\sigma_{\ln S}^2 + \sigma_{\ln L}^2}}$$

(A Signal-to-Noise Ratio)

The largest difficulties in this approach are

- *to find all sources of scatter and uncertainty,*
- *to estimate their variances and covariances, and*
- *to interpret the resulting reliability index I.*

The successful introduction of powerful statistical tools in quality engineering (6σ, DFSS) has inspired us to try something similar in reliability engineering.

*Using the presented simple second moment approach we aim at a **reliability tool-box** for engineering use.*

Variance estimates from prediction intervals

Statistical uncertainties originating from parameter estimates can be included in a prediction interval. Example: simple linear regression

$$y = a + bx + \varepsilon \qquad y_{pred}(x) = \hat{a} + \hat{b}x \pm t_{n-2,0.975}\, s\sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}}$$

$$\sigma^2_{\ln S} = \sum_j c_j^2 \sigma^2_{x_j} + \sum \mathrm{Cov} \approx$$

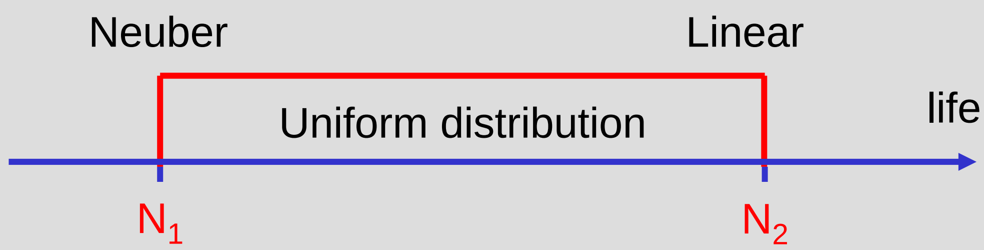
$$\frac{t^2_{0.975} s^2}{4}\left(1 + \frac{1}{n} + \frac{(x_1 - \bar{x}_1)^2}{\sum_i (x_{1,i} - \bar{x}_1)^2}\right) + \sum_{j>1} c_j^2 \hat{\sigma}^2_{x_j} + \sum \mathrm{Cov}$$

$$\frac{t^2_{0.975} s^2}{4}\left(1 + \frac{2}{n}\right) + \sum_{j>1} c_j^2 \hat{\sigma}^2_{x_j} + \sum \mathrm{Cov}$$

Variance estimates from engineering judgements

Example: model for plasticity.

- **Two models represent extreme cases of models, and the true behaviour is found somewhere in between.**
- **The *Neuber rule* and the *Linear rule* are extreme cases of plasticity models.**



Standard deviation of the uniform distribution

$$\delta = \frac{\ln N_2 - \ln N_1}{\sqrt{12}}$$

Conclusion

- **Model uncertainty:**

$$\delta = \frac{\ln 21 - \ln 1.77}{\sqrt{12}} = 0.72$$

The interpretation of the reliability index

$$I = \frac{\mu_{\ln(S)} - \mu_{\ln(L)}}{\sqrt{\sigma^2_{\ln S} + \sigma^2_{\ln L}}}$$

If the strength and load are lognormal, it is straightforward to use probabilities as reliability measures.

However, this is in general not the case, and for low percentiles the lognormal assumption may be dangerous.

There is a need for extra safety based on experience and judgements.

A possibility is to specify a limit

$$I > I_D$$

and let the limit be decided according to failure experience and consequences.

We are back to engineering judgement!

But, with the statistical approach the advantages of more strength tests, more accurate physical models, and more studies of users will be observable, and updating can be done in a rational way.