VMEA* in Practice

Lecture in the course *Failure Mode Avoidance*,
arranged by UTMIS & GMMC

Pär Johannesson, 23-Nov-2006

Goal: **Simple method for judging and taking into account variation and uncertainties in fatigue design.**

Pär Johannesson, FCC
Thomas Svensson, FCC
Leif Samuelsson, Volvo Aero
Jacques de Maré, Mathematical statistics, Chalmers
Bo Bergman, Quality Sciences, Chalmers

***VMEA = ”Variation Mode and Effect Analysis”**

Example: Volvo Aero

Low Pressure shaft

Life prediction	logarithmic life, ln(N)		
Type of scatter & uncertainty	scatter	uncertainty	total
Strength scatter			**0.38**
- Material, within shaft	0.15		
- Material, between shafts	0.29		
- Geometry	0.20		
Statistical uncertainty			**0.07**
- LCF-curve		0.07	
Model uncertainty			**0.84**
- LCF-curve		0.05	
- Mean stress model		0.30	
- Multi- to uni-axial		0.20	
- Plasticity		0.72	
- Stress analysis		0.24	
- Temperature		0	
Load scatter & uncertainty			**0.58**
- Service load, scatter	0.50		
- Service load, uncertainty		0.30	
Total	**0.63**	**0.90**	**1.10**

Monitoring of crack propagation during fatigue testing. The crack starts at an oil hole on the shaft.

Statistical approach vs. Safety factor

Advantages compared to classical safety factor method:

- Possible to get a measure of the risk of failure,
  e.g. failure probability or distance from failure mode.
- Possible to get a better understanding about the weakest link,
  i.e. where it is most profitable to put in more resources.
- Possible to relate the safety margin to the number of tests,
  i.e. the safety margin can be reduced by more tests.
- Possible to update model from failure reports,
  e.g. Bayesian updating.

Statistical approach

Problems to solve:

- The sources of scatter and uncertainties need to be identified and modelled by statistical measures.
- The different sources need to be combined into a total prediction uncertainty.
- An acceptable failure probability of distance from failure mode need to be decided.

Scatter & Uncertainty

We classify sources affecting the prediction uncertainties:

Scatter: (or aleatory uncertainty)
- The inherent random variation of the physical phenomenon of interest.
- Can not be reduced.
  (unless large changes are made of the produce or in the production)

Uncertainty: (or epistemic uncertainty)
- Caused by lack of knowledge or information.
- Can be reduced by better knowledge or more information.
  (e.g. better models or more data)

Ref: Melchers, R. (1999): *Structural Reliability Analysis and Prediction.* John Wiley & Sons, 2nd edition.

Different types of uncertainties

**There are many different types of (epistemic) uncertainties.
Here we make a distinction between two types:**

Statistical uncertainty:
- Depends on the limited amount of data available for estimating the model parameters.
- Can be reduced by for example more data, better estimation procedures, or prior information (engineering knowledge).

Model uncertainty:
- Depends on that our model does not perfectly reflect reality, e.g. when a complex phenomenon is modelled by a simple linear relation.
- Can be reduced by for example better or refined models.
  (However often more model parameters are needed. Model complexity!)

A simple model for the uncertainty in life predictions

Model and prediction error for logarithmic life:

$$\begin{aligned} \hat{e} &= \ln N - \ln \hat{N} \\ &= g(\Delta\varepsilon, \tilde{\mathbf{X}}) - f(\Delta\varepsilon, \hat{\boldsymbol{\theta}}, \hat{\mathbf{X}}) \\ &= X_1 + \ldots + X_p + Z_1 + \ldots + Z_q \end{aligned}$$

$g(\Delta\varepsilon, \tilde{\mathbf{X}})$	Actual relation for life.
$f(\Delta\varepsilon, \hat{\boldsymbol{\theta}}, \hat{\mathbf{X}})$	Model for life.
$\Delta\varepsilon$	Strain range, damage driving parameter.
$\boldsymbol{\theta} = (\theta_1, \ldots, \theta_r)$	Model parameters.
$\mathbf{X} = (X_1, \ldots, X_p)$	Scatters in log-life, zero mean and variance τ_k^2
$\mathbf{Z} = (Z_1, \ldots, Z_q)$	Uncertainties in log-life, zero mean and variance δ_k^2

Based on linearization and normal approximation.

Example: Model for life prediction

Model for logarithmic life: $\ln \hat{N} = f(\Delta\varepsilon, \hat{\boldsymbol{\theta}})$

Coffin-Manson's model:

$$\Delta\varepsilon = aN^{-b} \quad \Rightarrow \quad \ln N = f(\Delta\varepsilon, \boldsymbol{\theta}) = \frac{\ln a}{b} - \frac{1}{b} \cdot \ln \Delta\varepsilon = \theta_1 + \theta_2 \cdot \ln \Delta\varepsilon$$

- Linear regression.

Scatter, X_k:

- Material.
- Geometry.

Uncertainties, Z_k:

- Models for life (LCF-curve, plasticity, mean value correction, …).
- Estimation of model parameters.
- Calculation of stresses and strains.

Why log-life?

Two reasons:

1. **The logarithmic transformation of life implies that we measure the coefficient of variation for life.**
2. **The fatigue life distribution is often well approximated by a log-normal distribution.**

Standard deviation of log-life: $$Std[\ln N] \approx \frac{Std[N]}{E[N]}$$ **is approximately the coefficient of variation.**

Note: Natural logarithm ln (not the logarithm with base 10).

The interpretation of the natural logarithm is very practical for engineering applications, where uncertainties are often judged in percentage of variation.

Example: Volvo Aero

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- Service load, uncertainty		0.30	
Total	**0.55**	**0.90**	**1.05**

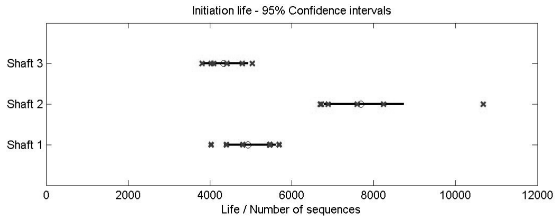
$$\delta_{tot} = \sqrt{\delta_1^2 + \delta_2^2 + \ldots + \delta_p^2 + \tau_1^2 + \tau_2^2 + \ldots + \tau_q^2}$$

Total prediction uncertainty

Example: Material scatter

Fatigue testing of Low Pressure shafts.

- **3 shafts, 6 cracks each.**
- **ANOVA (Analysis of variance)**

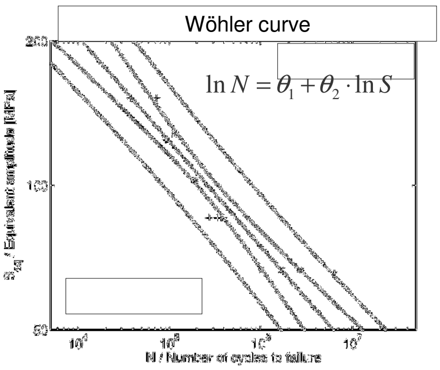


Monitoring of crack propagation during fatigue testing.
The crack starts at an oil hole.

Conclusion

- **Within shaft scatter:** $\tau = 0.15$
- **Between shafts scatter:** $\tau = 0.29$

Example: Statistical uncertainty



Wöhler curve – linear regression

- **Prediction uncertainty.**

$$s\sqrt{1+\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum_i (x_i-\bar{x})^2}} \approx s\sqrt{1+\frac{1}{n}+\frac{1}{n}} = s\sqrt{1+\frac{2}{n}}$$

Simplified formula

- **Model uncertainty, *r* parameters, *n* tests**

$$\delta \approx s\sqrt{\frac{r}{n}}$$

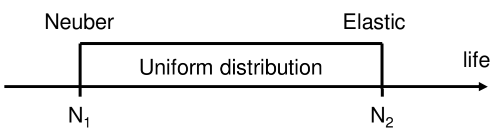
Conclusion – Volvo Aero Example

- **Statistical uncertainty:** $\delta = s\sqrt{\frac{r}{n}} = 0.15\sqrt{\frac{4}{20}} = 0.07$
- **Coffin-Manson with r=4 parameters based on n=20 tests, and s=0.15.**

Example: Model uncertainty

Model for Plasticity.

- **Principle: one model representing the least favourable case, and another representing the most favourable case.**
- **Two models represent extreme cases of models, and all other models predict lives somewhere in between.**
- **The Neuber rule and elastic calculation are extreme cases of models.**



Standard deviation of Uniform distribution

$$\delta = \frac{\ln N_2 - \ln N_1}{\sqrt{12}}$$

Conclusion

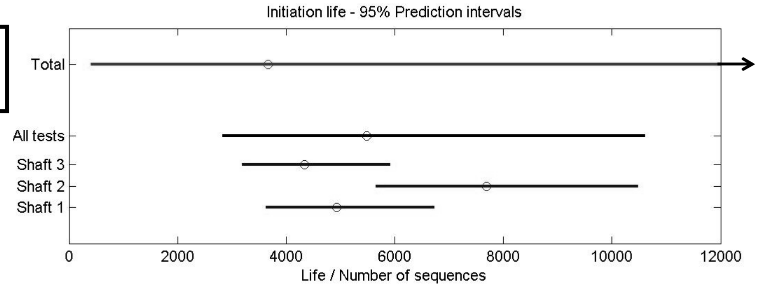
- **Model uncertainty:** $\delta = \dfrac{\ln 21 - \ln 1.77}{\sqrt{12}} = 0.72$

Prediction interval

Prediction interval based on total prediction uncertainty together with normal approximation:

$$\ln N = \ln \hat{N} \pm \lambda_p \cdot s \quad \Rightarrow \quad N = \hat{N} \cdot \exp(1 \pm \lambda_p \cdot s)$$

Example: Volvo Aero



Initiation life	Quantiles				
	0.1%	2.5%	50%	97.5%	99.9%
Life prediction	160	450	3 700	30 000	86 000

Safety factors based on prediction uncertainties

It is possible to define a safety factor in life, based on the prediction interval.

Safety factor = ”quotient between median life and a low quantile”

$$K_p = \frac{N_{0.5}}{N_p} \quad \Rightarrow \quad K_p = \exp(\ln N_{0.5} - \ln N_p) = \exp(\lambda_p \cdot s)$$

Safety factors for the Volvo Aero example:

- Failure risk of 2.5% (based on 95%- prediction interval) $K_{0.025} = 8.2$
- Failure risk of 1/1000 (based on 99.8% prediction interval) $K_{0.001} = 23$

Summary

Identify and quantify the sources of scatter and uncertainty

- **Scatter**: fatigue tests, tolerances, load, engineering judgement.
- **Statistical uncertainty**: model estimation, tests, approximations, experience.
- **Model uncertainty**: calculations, engineering judgement, round robin.

Calculate prediction uncertainty

- **Summarize the scatter and uncertainties in a table and calculate the total prediction uncertainty by Root Sum of Squares (RSS).**

Result

- **Prediction interval for life** (for use in design assessment).
- **Safety factor** (based on prediction uncertainty).
- **Identify the weakest link** (i.e. largest contribution to prediction uncertainty).

References & Acknowledgements

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References:

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2. Chakhunshvili, A. (2006): *Detecting, Identifying and Managing Sources of Variation in Production and Product Development*. PhD-thesis, Quality Science, Chalmers.
3. Johansson, P., Chakhunshvili, A., Barone, S., and Bergman, B. (2006): Variation mode and effect analysis: a practical tool for quality improvement. *Quality and Reliability Engineering International*. (in press)