Optimal control of vehicles via adaptive finite element methods

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Background

- Optimal control of the dynamics of heavy vehicles.
- Relevant for electronic stability control (ESC) (closed loop).
- Adaptive finite element methods.

Optimal Control of ODEs

Find states $y(t) \in \mathbb{R}^{d_1}$ and controls $u(t) \in \mathbb{R}^{d_2}$ which fulfill

$$\begin{aligned} \text{minimize} \quad & \mathcal{J}(y,u) = l(y(0), y(T)) + \int_0^T L(y(t), u(t))\, dt \\ \text{subject to} \quad & \dot{y}(t) = f(y(t), u(t)), \\ & I_0 y(0) = y_0, \ \ I_T y(T) = y_T. \end{aligned}$$

I_0,I_T are diagonal matrices with zeroes or ones on the diagonals.

Solution strategies

- Direct approach: discretize the equations and solve the optimization problem.
- Indirect approach: derive the necessary conditions for optimality, discretize and solve.
- We use the indirect approach.

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Necessary Conditions for an Optimum

Variational Calculus

Hamiltonian: $H(y, \lambda, u) = L(y, u) + \lambda^{\mathrm{T}} f(y, u)$

The optimal y and u satisfy

$$\dot{y} = \frac{\partial H}{\partial \lambda} = f(y, u),$$

$$\dot{\lambda} = -\frac{\partial H}{\partial y} = \frac{\partial L}{\partial y} - \left(\frac{\partial f}{\partial y}\right)^{\mathrm{T}} \lambda,$$

$$0 = \frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \lambda^{\mathrm{T}} \frac{\partial f}{\partial u},$$

$$I_0 y(0) = y_0, \quad I_T y(T) = y_T,$$

$$(I - I_0)\lambda(0) = \lambda_0, \quad (I - I_T)\lambda(T) = \lambda_T.$$

This is a Boundary Value Problem for a system of DAE.

Numerical Methods

- Shooting methods.
- Collocation methods.
- We use the Finite Element Method.
- The finite element method is based on a variational formulation and piecewise polynomials.
- The finite element method is well suited for mathematical analysis.
- Admits error control and adaptive mesh generation based on *a posteriori* error estimates.

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Results so far

- *Using an adaptive FEM to determine the optimal control of a vehicle during a collision avoidance manoeuvre*, Proceedings of SIMS2007
- Application of a standard adaptive finite element method to an optimal control problem from vehicle dynamics.
- Combine $x = (y, \lambda)$, eliminate u. ODE: $\dot{x} = g(x)$
- A posteriori error estimate for an arbitrary linear functional:

$$|G(x - x_h)| \leq \sum_{n=1}^{N} R_n \omega_n$$

2007

Results so far

- *The dual weighted residuals approach to optimal control of ordinary differential equations,* Preprint 2008:2.
- The Dual Weighted Residuals methodology: keep y, λ, u.
- Take advantage of the variational structure of the equations.
- A posteriori error estimate for the cost functional

$$|\mathcal{J}(y,u) - \mathcal{J}(y_h,u_h)| \leq \sum_{n=1}^{N} R_n^y \omega_n^\lambda + R_n^\lambda \omega_n^y + R_n^u \omega_n^u$$

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Results so far

- Implemented the above algorithms in Matlab.
- The implementation is not completely general.

Future Work 1

Extend the theory

- Efficient solution of the resulting linear/nonlinear algebraic equations.
- Constraints on controls and states (Pontryagin max principle or slack variables).
- A priori error estimates.

Future Work 2

Implementation

- Non-linear problems
- Constraints on controls and states
- More advanced models from vehicle dynamics

Midterm evaluation

- The present 2 publications: licentiate thesis of Karin Kraft.
- One more publication: realistic models from vehicle dynamics.
- I hope that we will have demonstrated that adaptive FEM is useful for optimal control of vehicles.
- (Karin Kraft, PhD 2009, expected.)

Questions, discussion

- Choice of programming language (Matlab, C++, Fortran or other)
- Include constraints on controls and states
- More interaction with GMMC
- Parameter estimation
- Stochastic control

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ r \\ \psi \\ X \\ Y \end{bmatrix},$$

$$\dot{y} = \begin{bmatrix} \dot{V}_X \\ \dot{V}_Y \\ \dot{r} \\ \dot{\psi} \\ \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21}V_y + a_{22}r + bf_1\delta_f + br_1\delta_r \\ a_{31}V_y + a_{32}r + bf_2\delta_f + br_2\delta_r + r_{\text{brake}} \\ r \\ V_x \\ V_y \end{bmatrix}$$

$$l_0 y(0) = \begin{bmatrix} 25 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$J(y,u) = \int_0^T \left(Y^2 + r^2 + \psi^2 + \delta_f^2\right) dt = \int_0^T \left(\|y\|_Q^2 + \|u\|_R^2\right) dt.$$

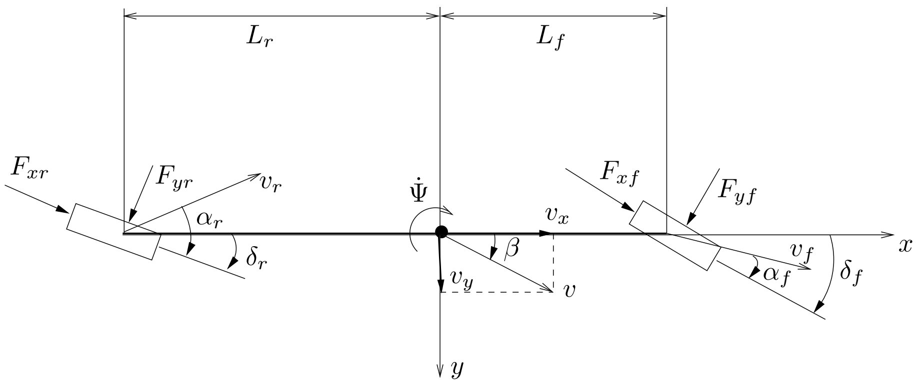

Figure 3: The bicycle model, which is used to derive a model of the dynamics of a vehicle. The rectangles represent the wheels of the bicycle, and the dot marks the center of gravity around which the angular velocity is computed.

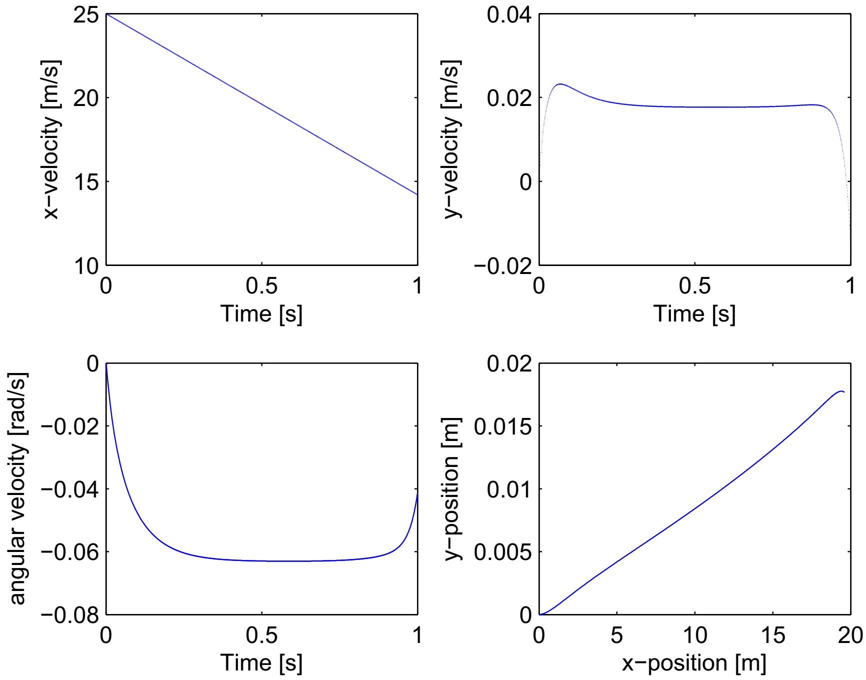

Figure 4: The optimal states. The last picture shows the optimal track.