Nanoscience - Nanomechanics (MCC026)

Exercise - 3
November 23rd 2018
Witlef Wieczorek (*witlef.wieczorek@chalmers.se*)

1 Mechanical transduction

Question 1: Transduction methods

(i) Name two different methods of how to transduce (i.e. read-out) a mechanical resonator and explain their basic working principle.

2 Mechanical force noise

White thermomechanical force noise acting on a mechanical resonator with mass m and damping γ_m at a temperature T is given as

$$S_{FF}^{(\mathrm{tm})} = 4k_B T m \gamma_m, \tag{1}$$

with k_b Boltzman's constant and $S_{FF}^{(\mathrm{tm})}$ is given in units N^2/Hz.

One also has to read-out a mechanical resonator in order to infer the signal. The minimal measurement impression noise on resonance ω_0 is given as

$$S_{FF}^{(\mathrm{imp})}(\omega_0) = \frac{m^2}{|\chi(\omega_0)|^2} S_{xx}(\omega_0), \tag{2}$$

with the mechanical susceptibility $\chi(\omega) = ((\omega_0^2 - \omega^2) - i\gamma_m\omega)^{-1}$ and $S_{xx}(\omega_0) = x_{\mathrm{zpf}}^2/\gamma_m$ with the mechanical zero-point motion $x_{\mathrm{zpf}} = \sqrt{\hbar/(m\omega_m)}$ and $\hbar = h/(2\pi)$ with h as Planck's constant.

Question 2: Which platform is better suited for force sensing?
Consider two methods of detecting force using a mechanical resonator based on

- a carbon nanotube as in S. L. Bonis et al. [Nano Letters 18, 5324 (2018)] with a mass of 8.6 attogram, a quality factor $Q_m = 6.27 \cdot 10^5$, resonance frequency $f_m = 29.8\,\mathrm{MHz}$ at a temperature of 120 mK or,
- a levitated magnetic sphere made from Nd2Fe14B as in J. Prat-Camps et al. [Phys. Rev. Appl. 8, 034002 (2017)] with a radius of 100 nm, a density of $7300\mathrm{kg/m}^3$, a quality factor $Q_m = 10^9$, resonance frequency $f_m = 200\,\mathrm{Hz}$ at a temperature of 1K.

(i) Calculate the expected thermal force noise $S_F^{(\mathrm{tm})} = \sqrt{S_{FF}^{(\mathrm{tm})}}$ (in units $\mathrm{N}/\sqrt{\mathrm{Hz}}$) for each of the methods. Which method is more sensitive?

(ii) Calculate the expected minimal imprecision noise at resonance $S_F^{(\mathrm{imp})} = \sqrt{S_{FF}^{(\mathrm{imp})}}$ (in units $\mathrm{N}/\sqrt{\mathrm{Hz}}$) for each of the methods. Why is the imprecision noise smaller than the thermal force noise? What would one need to do in order to decrease the thermal force noise?

(iii) Name at least one other noise source (besides thermomechanical and imprecision noise) that should be considered when calculating the minimal sensitivity of a mechanical resonator to an external force.

(iv) Why is the quality factor of the levitated magnet so much larger than the one of the nanotube?

(v) Calculate the attractive gravitational force between a human on Earth and a human on the moon (neglect all other masses). Compare this force to the force sensitivities of a nanotube and a levitated magnet when measured for 1 second.