Nanoscience - Nanomechanics (MCC026)

Exercise - 1
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1 Eigenmodes of beams

1.1 Doubly-clamped beam

In the lecture we have derived the eigenfrequencies of a doubly-clamped beam of length L, cross-sectional area A, density ρ, Young's modulus E and geometric moment of inertia I_y:

$$\omega_n = \beta_n^2 \sqrt{\frac{E I_y}{\rho A}}, \tag{1}$$

with the solutions of the boundary conditions for a doubly-clamped beam given as $\lambda_n = \beta_n L \in \{4.73, 7.853, 10.9956, ...\}$.

Question 1: Eigenfrequencies of a suspended Carbon nanotube
In the work of S. L. Bonis et al. [Nano Letters 18, 5324 (2018)] a 1.3 μm long Carbon nanotube with Young's modulus of 1 TPa is suspended, see Fig. 1.

(i) Calculate the eigenfrequencies of the first three eigenmodes of this nanotube. Assume that the nanotube has a diameter of 2.74 nm, its density can be taken as $\rho = 1120\,\mathrm{kg/m^3}$ and the geometric moment of inertia for a circular cross section of radius r is $I_y = \frac{\pi}{4} r^4$.

(ii) Compare your calculated eigenfrequencies to the actually observed ones, see Fig. 2. How would you explain the observed discrepancies?

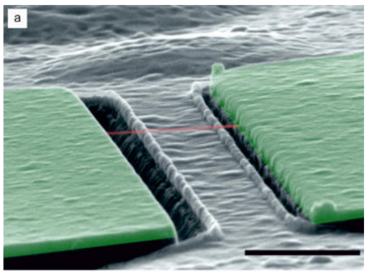

Figure 1: Suspended carbon nanotube. The scale bar is 1 μm, from S. L. Bonis et al. [Nano Letters 18, 5324 (2018)]

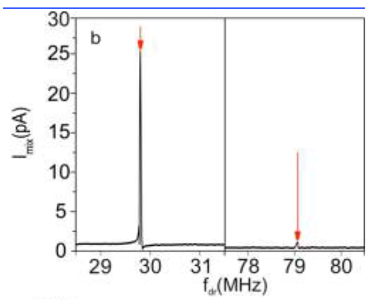

Figure 2: Frequencies of the Carbon nanotube are marked with an arrow, from S. L. Bonis et al. [Nano Letters 18, 5324 (2018)]

1.2 Tensile strained membrane

In the lecture you saw that tensile strain can be used to increase the frequency of a mechanical resonator compared to its unstrained counterpart. In case of a strained membrane of area $A = l \cdot w$, one obtains the resonant frequencies as

$$\omega_{mn} = \pi \sqrt{\frac{\sigma}{\rho}} \sqrt{\left(\frac{n}{l}\right)^2 + \left(\frac{m}{w}\right)^2}, \tag{2}$$

with tensile strain σ.

Question 2: Strained membrane
Look at Fig. 3. The noise power spectrum of a rectangular, tensile strained InGaP membrane with length $l = 0.92\,\text{mm}$ and width $w = 0.97\,\text{mm}$, density $\rho = 4470\,\text{kg/m}^3$ and strain of $160\,\text{MPa}$ is shown.

(i) Identify the eigenmodes of the observed frequencies at room temperature, i.e., mode (i, j) with frequency f_{ij} for modes $(1,1)$, $(1,2)$, $(2,1)$, $(1,3)$, $(3,1)$, $(1,4)$, $(4,1)$, $(2,2)$, $(2,3)$, $(3,2)$, $(2,4)$, $(4,2)$, $(3,3)$, $(3,4)$, $(4,3)$.

(ii) Why do the eigenfrequencies shift when lowering the temperature to 17 K?

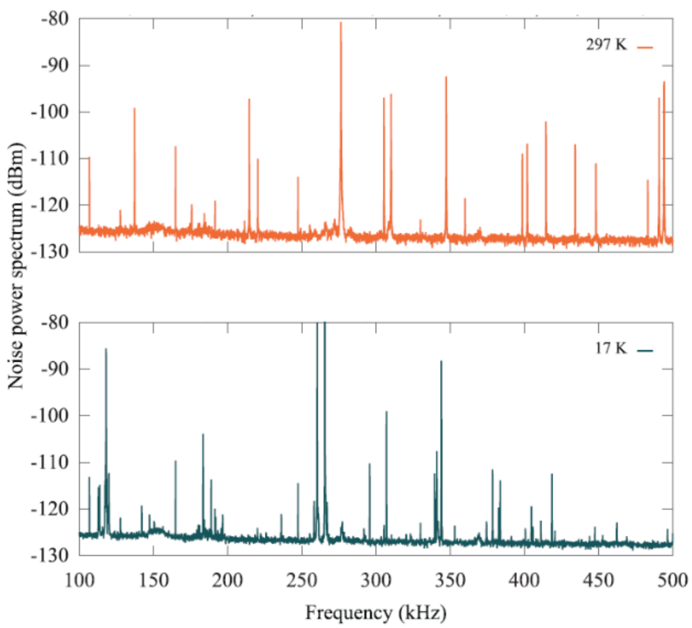

Figure 3: Observed frequencies versus laser power for a tensile strained, square membrane, from G. D. Cole et al. [Appl. Phys. Lett. 104, 201908 (2014)]

1.3 Singly-clamped beam

In the lecture, we have derived the eigenfrequencies and mode functions in case of a doubly clamped beam. We have started with the general Euler-Bernoulli equation

$$\rho A \frac{\partial^2 u(x,t)}{\partial t^2} + E I_y \frac{\partial^4 u(x,t)}{\partial x^4} = 0 \tag{3}$$

using the Ansatz for separating time from spatial dependence $u(x,t) = \sum_{n=1}^{\infty} U_n(x) \cos(\omega t)$ and with the spatial part given as $U_n(x) = a_n \cos(\beta_n x) + b_n \sin(\beta_n x) + c_n \sinh(\beta_n x) + d_n \sinh(\beta_n x)$.

Question 3: Singly-clamped beam (takes time to calculate)

(i) Derive the eigenfrequencies ω_n and mode functions $U_n(x)$ for a singly-clamped beam of length L. A singly-clamped beam is clamped on one side ($U_n(0) = \frac{\partial}{\partial x} U_n(0) = 0$), whereas it is free to move on its other end (then you use the following boundary conditions for the curvature and momentum of the beam: $\frac{\partial^2}{\partial x^2} U_n(L) = \frac{\partial^3}{\partial x^3} U_n(L) = 0$). For calculating the zero-crossings of the determinant, use a numerical software such as Mathematica or Matlab.

2 Lumped element model

In the lecture, we have seen that we can describe the temporal evolution of the mode amplitude $x(t)$ through a harmonic oscillator equation:

$$\ddot{x}(t) + \gamma_m \dot{x}(t) + \omega_m^2 x(t) = \frac{1}{m} \sum_i F_i(t), \tag{4}$$

where $\omega_m^2 = k/m$ is the eigenfrequency of the mechanical resonator, γ_m is its damping, m its mass, k its spring constant and F_i an external force.

Question 4: RCL-equivalence

The above equation has a formal analogy with an electronic circuit, where a resistor R, a capacitor C and an inductor L are connected in series. Assume that you have such an RCL circuit in series with a voltage source $V(t)$ connected.

(i) Derive the dynamic equation for the current $I(t)$ through such an RCL circuit.

(ii) What does ω_m, γ_m and $F_i(t)$ correspond to in your derived equation when you formally identify $x(t)$ with $I(t)$?