Homework on Nanooptics

Formulas

Induced dipole moment in a particle when it is subject to an applied electric field with strength E_0

$$p = \alpha E_0$$

Polarizability of a spherical particle with radius r much smaller than the wavelength of light. The particle is made of a material with dielectric constant ε and situated in a homogeneous medium with dielectric constant ε_m

$$\alpha = 4\pi r^3 \, {}^{\varepsilon - \varepsilon_m}/_{\varepsilon + 2\varepsilon_m}$$

Absorption and scattering cross-sections ($k = 2\pi/\lambda$, where λ is the light wavelength in the medium surrounding the particle):

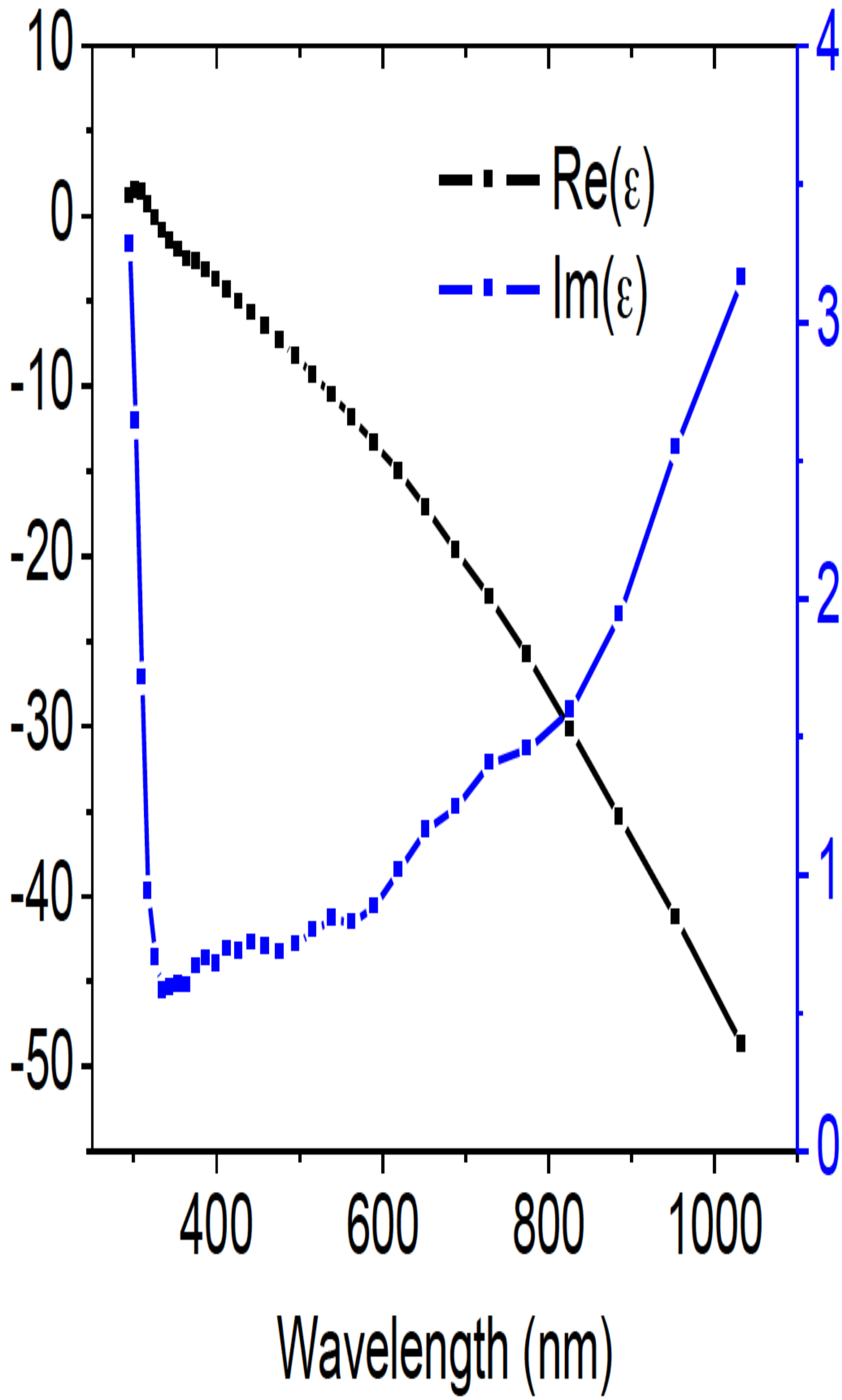
$$\sigma_{abs} = k\mathrm{Im}[\alpha]; \, \sigma_{scat} = {}^{k^4|\alpha|^2}/_{6\pi}$$

Frequency dependent dielectric function of a free-electron metal with plasma frequency ω_p and electron "collision rate" $1/\tau$ according to Paul Drude

$$\varepsilon(\omega) = 1 - \frac{{\omega_p}^2}{\omega^2 + i\omega/\tau}$$

Questions

1) How do one "beat" the diffraction limit in optical microscopy based on a) NSOM, b) PALM/STORM, c) STED. Describe the basic principles behind the techniques based on simplified drawings (use Wikipedia etc to learn more if needed). What are the pros and cons of the techniques compared to each other and compared to classical optical microscopy (e.g. confocal fluorescence microscopy).
2) Show that the localized surface plasmon (LSP) resonance frequency of a small "free-electron" metal particle in vacuum is approximately given by $\omega_{LSP} = {}^{\omega_p}/_{\sqrt{3}}$
3) The figure below shows the real and imaginary parts of the dielectric constant of silver as a function of vacuum wavelength. For the case of a small silver particle, estimate the LSPR wavelength for the case when the particle is immersed in air (n≈1), water (n≈1.33) and glass (n≈1.5).
4) Estimate the maximum absorption and scattering cross-sections for a silver particle with a radius of 50 nm for the case when it surrounded by glass. Give the result as a fraction of the geometrical cross-section of the particle and compare the result to the case of 1) a water droplet of the same size in air, 2) an air bubble of the same size in glass.



10
0
-10
-20
-30
-40
-50
4
3
2
1
0
Re(ε)
Im(ε)
400
600
800
1000
Wavelength (nm)