Nanoscience MCC026, Quantum Transport Theory
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Exercise 6: Quantum Hall effect : "Edge states" in a harmonic potential

Consider a two-dimensional $(x - y)$ plane with a strong perpendicular magnetic field in z-direction. Electrons in the plane are described by the Hamiltonian

$$H = \frac{1}{2m}(\boldsymbol{p} + e\boldsymbol{A})^2 \tag{1}$$

In the lecture, it was shown that this leads to an energy spectrum with so-called Landau levels in the following way :

Using the Hamiltonian of Eq. (1) and the Landau gauge for the vector potential, $\boldsymbol{A} = (-By, 0, 0)$, together with the ansatz

$$\Psi(x, y) = e^{ikx}\chi(y) \tag{2}$$

one can write a harmonic oscillator equation for $\chi(y)$. This has the form

$$\left[\frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 (y - y_0)^2\right]\chi(y) = E\chi(y) \tag{3}$$

with the cyclotron frequency $\omega = \frac{eB}{m}$ and the guiding center $y_0 = \frac{\hbar k}{eB}$. The harmonic oscillator is known to have the spectrum $E = \hbar\omega\left(n + \frac{1}{2}\right)$.

In this exercise, a simplified example for the possible occurrence of current carrying, chiral edge states due to a confinement potential is studied. We therefore consider the (not so realistic) example of a harmonic confinement potential in y-direction :

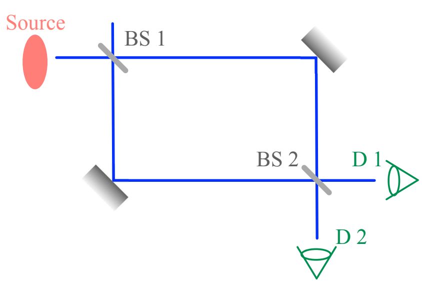
$$V(y) = \frac{1}{2}m\omega_0^2 y^2 \tag{4}$$

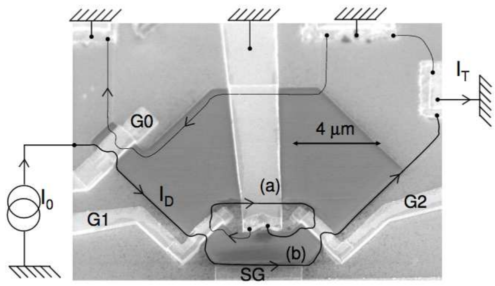
a.) Add the potential given in Eq. (4) to the Hamiltonian. Now, derive the spectrum (using the information about the harmonic oscillator equation given above) and show that a dependence on the guiding center $y_0 = \hbar k/eB$ emerges in the energy spectrum.

b.) Calculate the resulting non-vanishing velocity of electrons in the plane, $v = \frac{1}{\hbar}\frac{\partial E}{\partial k}$.

c.) Explain, why this potential can however not be used to create *edge* states.

Exercise 7: Electronic Mach-Zehnder interferometer





This exercise deals with an electronic Mach-Zehnder interferometer – the electronic analog of the setup known from optics as shown on the left hand side in the sketch above. It can be realized with the help of chiral quantum Hall edge states taking the role of waveguides. An electron micrograph of this is shown on the right handside of the figure (P. Roulleau et al. : Phys. Rev. B **76**, 161309 (2007)). The only ingredient from the quantum Hall physics that has to be considered here is the chirality of the (single!) transport channel.[1]

We now assume that all contacts are grounded except for the source (indicated in red in the sketch), such that $\mu_{\text{source}} = \mu_0 - eV$. Assuming furthermore that the current is measured at the detector D1, we only need to calculate the transmission probability from the source to the detector D1 in order to calculate the current.

A wave coming in from the source can either reach D1 by being transmitted at the beamsplitter BS1, traveling along the upper path and being reflected at beamsplitter BS2, or it can be reflected at BS1, travel along the lower path and reach detector D1 after being transmitted at beamsplitter BS2.

a.) Write down the transmission probability of a wave from the source to the detector D1. Use that the reflection and transmission amplitudes of the two beam splitters are given by energy-independent r_1, t_1 and r_2, t_2. Take the upper (lower) path of the interferometer to have length L_{up} (L_{down}); these parameters enter into the dynamic phase. The difference of geometric phases acquired by the wave function on the two paths is determined by the magnetic flux Φ penetrating the interferometer.

b.) To get the energy-dependence of the transmission probability, entering via the phases $ikL_{\text{up/down}}$, use that the spectrum is quadratic, $E = \hbar^2k^2/2m$. Linearize the energy dependence around the Fermi energy $E_{\text{F}} = \mu_0$ at which the wave-vector takes the value $k = k_{\text{F}}$. Use that the product of length $L_{\text{up/down}}$ and the inverse of the Fermi velocity v_{F} defines a time-scale, which you can call $\tau_{\text{up/down}}$.

c.) Using the Landauer-Büttiker formula, calculate the conductance in the limit of zero temperature

$$I = -\frac{e}{h}\int dE\; T(E)\left(f_{\text{source}}(E) - f_{\text{detector}}(E)\right) \quad \xrightarrow{\text{zero temperature}} \quad G = \frac{e^2}{h}T(\mu_0) \tag{5}$$

for the interferometer, with the transmission probability calculated in the previous parts of the exercise.

d.) With the help of your result from c.), explain/discuss the experimental data shown below (Y. Ji et al. : Nature **422**, 415 (2003)). For the two curves that are shown here, two different parameters are changed in two independent experiments :[2] the blue, empty circles refer to the lower x-axis. Since the magnetic field slowly changes in time in this experiment, the time on the axis simply stands for changes in the magnetic field. The red filled circles refer to the upper x-axis. Changes in the gate-potential V_{MG} induce changes in the path lengths of the interferometer and thereby also of the area they enclose.

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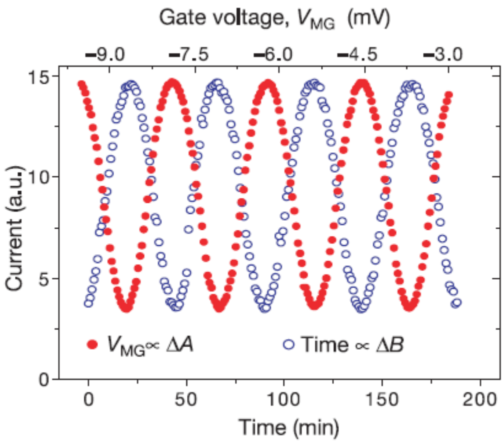
1. It thereby constitutes a great simplification for your calculation, since there are no multiple scattering trajectories to be considered, compared to the Aharonov-Bohm ring discussed in the lecture.

2. The two curves are shifted to each other in a way to make the oscillations visible. The relative shift between them has no physical meaning and does not need to be discussed.



e.) Now go to the more general case of large bias and non-zero temperature and write down the expression for the full current, see Eq.(5). How do you qualitatively expect the temperature and bias to influence the amplitude of the magnetic-field dependent oscillations of the current ? Motivate your expectations.