Nanoscience MCC026, Quantum Transport Theory
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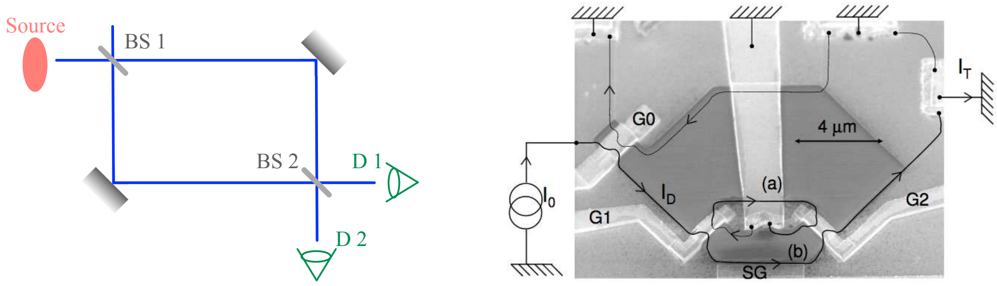
Exercise 8: Electronic Mach-Zehnder interferometer



This exercise deals with an electronic Mach-Zehnder interferometer – the electronic analog of the setup known from optics as shown on the left hand side in the sketch above. It can be realized with the help of chiral quantum Hall edge states taking the role of waveguides. An electron micrograph of this is shown on the right handside of the figure (P. Roulleau et al. : Phys. Rev. B **76**, 161309 (2007)). The only ingredient from the quantum Hall physics that has to be considered here is the chirality of the (single!) transport channel.[1]

We now assume that all contacts are grounded except for the source (indicated in red in the sketch), such that $\mu_{\text{source}} = \mu_0 - eV$. Assuming furthermore that the current is measured at the detector D1, we only need to calculate the transmission probability from the source to the detector D1 in order to calculate the current.

A wave coming in from the source can either reach D1 by being transmitted at the beamsplitter BS1, traveling along the upper path and being reflected at beamsplitter BS2, or it can be reflected at BS1, travel along the lower path and reach detector D1 after being transmitted at beamsplitter BS2.

a.) Write down the transmission probability of a wave from the source to the detector D1. Use that the reflection and transmission amplitudes of the two beam splitters are given by energy-independent r_1, t_1 and r_2, t_2. Take the upper (lower) path of the interferometer to have length L_{up} (L_{down}); these parameters enter into the dynamic phase. The difference of geometric phases acquired by the wave function on the two paths is determined by the magnetic flux Φ penetrating the interferometer.

b.) To get the energy-dependence of the transmission probability, entering via the phases $ikL_{\text{up/down}}$, use that the spectrum is quadratic, $E = \hbar^2 k^2/2m$. Linearize the energy dependence around the Fermi energy E_{F} at which the wave-vector takes the value $k = k_{\text{F}}$. Use that the product of length $L_{\text{up/down}}$ and the inverse of the Fermi velocity v_{F} defines a time-scale, which you can call $\tau_{\text{up/down}}$.

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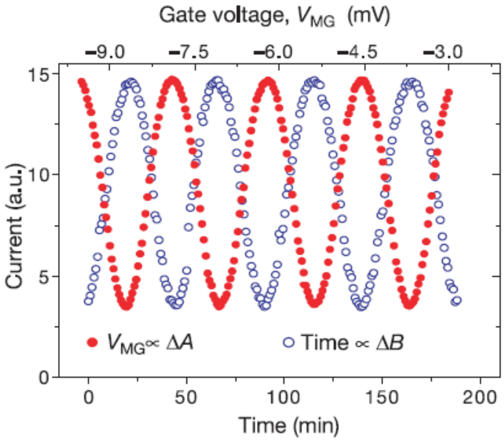
1. It thereby constitutes a great simplification for your calculation, since there are no multiple scattering trajectories to be considered, compared to the Aharonov-Bohm ring discussed in the lecture.

c.) Using the Landauer-Büttiker formula, calculate the conductance of the interferometer in the limit of zero temperature.

d.) With the help of your result from c.), explain/discuss the experimental data shown below (Y. Ji et al. : Nature **422**, 415 (2003)). For the two curves that are shown here, two different parameters are changed : the blue, empty circles refer to the lower x-axis. Since the magnetic field slowly changes in time in this experiment, the time on the axis simply stands for changes in the magnetic field. The red filled circles refer to the upper x-axis. Changes in the gate-potential V_{MG} induce changes in the path lengths of the interferometer and thereby also of the area they enclose.



e.*) Now go to the more general case of large bias and non-zero temperature and write down the expression for the full current. How will temperature and bias influence the amplitude of the magnetic-field dependent oscillations of the current ? Support your ideas by numerically integrating and plotting the current.