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A linearised solver for wind-farm flows

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Linearized simulation of flow over wind farms and complex terrains

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The flow over complex terrains and wind farms is estimated here by numerically solving the linearized Navier-Stokes equations. The equations are linearized around the unperturbed incoming wind profile, here assumed logarithmic. The Boussinesq approximation is applied and the roughness is parameterized with a log-law relationship.
Introduction

- The site-assessment and layout-optimisation process are two important parts of every wind-energy project.
- A complete experimental assessment is not always feasible.
- Numerical simulations can nowadays predict most of the relevant features of the flow over complex terrains.
- Are they fast enough for optimization purposes?
- What about wind-turbine wake models?
A variety of models exist to assess the AE

How many of the industrial models can account correctly for terrain or stratification effects?
A possible alternative: Linear methods

- Besides some scatter, linear methods have a similar error
- Obviously they are not expected to be accurate, but they provide the leading flow features
- Linear methods are already used (WAsP, FUGA, ... , more based on wake models)

Corbett et al. (2014), EWEA
Linearised approach

- The equations are linearised around the incoming base flow (assumed to be parallel) while the perturbation is provided by the distributed body forces \((F_1,F_2,F_3=0)\)
- Inviscid flow, homogeneous incoming flow

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

\[
U_0 \frac{\partial u}{\partial x} = - \frac{\partial p}{\partial x} + F_1
\]

\[
U_0 \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + F_2
\]

\[
U_0 \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z}
\]

\[
\nabla^2 p = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y}
\]

Analytically solvable

\[
\mathbf{U} = [U_0 + u, v, w]
\]

\(u, v, w \ll U_0\)
Linear methods

\[ U_j \frac{\partial U_i}{\partial x_j} = - \frac{\partial P}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 U_i}{\partial x^2_j} - \frac{\partial u'_i u'_j}{\partial x_j} + F_i \]

NS-equations

\[ -u'_i u'_j = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} k \]

Closure hyp.

\[ \nu_t = \kappa z \]

Linearisation assumption

\[ U = [U_0(z) + u, v, w] \]

\[ u, v, w \ll U_0 \]

\[ U_0 \frac{\partial u}{\partial x} + \frac{\partial U_0}{\partial z} w = - \frac{\partial \Pi}{\partial x} + \left( \frac{1}{Re} + \nu_t \right) \nabla^2 u + \frac{d\nu_t}{dz} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) + F_1 \]

The turbines/terrain generate a small perturbation to the base flow
Final set of linear equations (terrain + turbines)

\[ \xi = x \]
\[ \gamma = y \]
\[ \eta = z - H(x, y) \]

\[ w = H_\xi U_0 + \tilde{w} \]

A coordinate mapping is introduced to account for terrain elevation.

The linear NS equations can be manipulated to get 1 equation for 1 variable:

\[ U_0 \nabla^2 \frac{\partial \tilde{w}}{\partial \xi} - \frac{d^2 U_0}{d\eta^2} \frac{\partial \tilde{w}}{\partial \xi} - \left( \frac{1}{L^+} + \nu_t \right) \nabla^4 \tilde{w} - 2 \frac{d\nu_t}{d\eta} \nabla^2 \frac{\partial \tilde{w}}{\partial \eta} = -U_0^2 \nabla^2 H_\xi - \frac{\partial}{\partial \eta} \left( \frac{\partial F_1}{\partial \xi} + \frac{\partial F_2}{\partial \gamma} \right) \]

- **Advection**
- **Viscous and turbulent transport**
- **Displacement imparted by the terrain \( z=H(x,y) \)**
- **Forest or Wind turbines body forces**

No pressure is involved....
Solution method

- Fourier transform in the horizontal plane
- Chebyshev polynomials in the vertical (non-homogeneous) direction
- The PDEs become many linear algebraic problems \((Ax=B)\), one for each wave number
- Easy to parallelise, very fast and numerically accurate
- Turbines are simulated as actuator disks

\[ F_1 = -\frac{T}{V} = -\alpha(U_0 + u)^2 \]

\[ T = \frac{1}{2} \rho U_\infty^2 A_d C_T \]

\[ \alpha \approx \frac{C_T}{2\delta} \left( \frac{U_\infty}{U_d} \right)^2 \]

- An iterative approach is used to estimate the body forces
Figure 1. Velocity statistics for the simulated forest case. (a) $0.4U/u_*$, (b) $-1.3\overline{u'w'}/u_*^2$. Circles indicate LES data for $LAI = 2$. Solid black lines indicate the corresponding linearised model results while the grey lines indicate the non-linear RANS simulation data. The vertical dashed lines indicate the $x$-positions where the quantities are evaluated, while the curved dashed lines in part (a) indicate the undisturbed wind profile.
Figure 2. Comparison between the experimental data of [16] (circles) against the linearised model result (solid line) at the positions (a) $x/H_0 = -2.5$, (b) $x/H_0 = -1.25$, (c) $x/H_0 = 0$, (d) $x/H_0 = 1.25$ and (e) $x/H_0 = 2.5$. The dashed line indicates the unperturbed wind profile.
Wind farm over a hill

Figure 4. Streamwise velocity on a surface at constant height, $\eta = z_{hub}/D$, for the hill case (a), the farm case (b) and the hill+farm case (c). (Online version in colour.)
Figure 7. Streamwise velocity on a surface in the middle of the farm, $\gamma = y/D = 0$, for the hill case (a), the farm case (b) and the hill + farm case (c). (Online version in colour.)
Figure 6. Spanwise velocity on a surface at constant height, $\eta = z_{\text{hub}}/D$, for the hill case (a), the farm case (b) and the hill+farm case (c). (Online version in colour.)
Validation of the farm case (upstream blockage)

- Good agreement between the model and the wind-tunnel data
- Upstream effect of the hill
Small farm over complex terrain

ORFEUS  WindSim

AE
Lillgrund wind farm (Vattenfall)
URANS (in progress…)

- With a turbulent inlet, it is possible to simulate also unsteady conditions
- Wake starts to meander
Conclusions and future outlooks

- The linearized model seems to be a useful tool able to account for forests, hills and turbines in more efficient ways than FUGA and more general than WAsP.
- There are many drawbacks with the linear approximation.
- The power production is generally underestimated.
- Unsteady simulations are feasible (wake meandering).
Thank you for your attention